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6. AUTHOR(S)
Thomas Murphey and Jason Hinkle

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13. ABSTRACT (Maximum 200 words)
Monolithically Elastically Deployable Latticed Structures (MEDLS) based on rod-like elements were studied. Analy optimization tools were developed and used to calculate the dimensions of lightest weight trusses subjected to bei strength and stiffness requirements. This investigation revealed that even with element straightness imperfection effects included, gossamer space structures (those with high stiffness and low strength requirements) optimally er very slender elements. Gossamer space structures (support structures for membrane optics, solar sails, sunshielc and mesh/wire antennas) have optimal slendernesses as high as 400, an order of magnitude greater than contemporary structures. Several structural architectures were considered and evaluated using non-linear analyse elastic packaging geometry and performance. The Single Elastic Loop Folding (SELF) method was identified as a efficient method to package all conceived truss-like architectures. A single circular loop is formed in all members, effectively scaling the truss node positions without global truss rotations. SELF requires element slendernesses >: for a high strength graphite/epoxy material system, restricting it to trusses in gossamer applications. MEDLS are ic suited to large gossamer deployable structures and can provide the space community with support structures that as lightweight as the membranes, meshes and wires they are intended to support.

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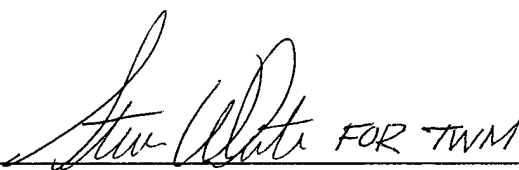# Innovative Space Materials and Structures

Phase I Final Report

for the

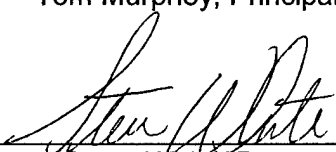# Defense Advanced Research Projects Agency (DARPA)
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Tom Murphey, Principal Investigator

Approved by: _____
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Table of Contents

Figures

Table of Contents (cont.)

Tables

Appendices

Innovative Space Materials and Structures
SBIR Phase I Final Report

1.0 INTRODUCTION

ABLE Engineering, Inc. (ABLE) has been under contract through the DARPA Small Business Innovative Research (SBIR) Phase I Program (Contract #DAAH01-03-C-R070) to investigate, develop and demonstrate the feasibility of a new structural architecture for deployable space structures: Monolithic Elastically Deployable Lattice Structures (MEDLS). MEDLS has the potential to offer order-of-magnitude reductions in structural mass over current deployable technologies. The architecture incorporates two key features. First, higher than state-of-the-art orders of structural hierarchy can be incorporated into MEDLS. Preliminary analyses have shown that highly hierarchical structures are extremely efficient for a wide range of applications. Second, elastic strain energy deployment is used to eliminate the parasitic mass and complexity associated with other deployable systems (such as articulated, inflatable and shape memory systems). In MEDLS, the same material that provides for structural stiffness also provides the deployment motivation and latching strength. Further, the elimination of joints enhances dimensional repeatability and stability and minimizes dead-band in the deployed structure.

A primary objective of this Phase I study was to identify and characterize monolithic deployable truss architectures that are conducive to efficient packaging by means of elastic material straining. To meet this objective, elastically deployable-latticed structures based on rod-like flexible elements were studied in two independent analytical investigations. In the first, robust optimization tools were developed and used to calculate the dimensions of lightest weight trusses subjected to bending strength and stiffness requirements. This investigation revealed that even with element straightness imperfection effects included, gossamer space structures (those with high stiffness and low strength requirements) optimally employ very slender elements. Gossamer space structures (structures to which MEDLS technology is directly applicable, such as support structures for membrane optics, satellite decoy support structures, solar sails, sunshields and mesh/wire antennas) have optimal slenderness (element length/diameter ratio, L/d) as high as 400. This is an order of magnitude greater than contemporary space-based deployable structures. In the second study, several structural architectures were considered and evaluated using non-linear finite element analyses for elastic packaging geometry and performance. The Single Elastic Loop Folding (SELF) method was identified and developed as a feasible and efficient method to

package many conceived truss-like architectures. In this unique implementation to MEDLS, a single circular loop is elastically formed in all members, effectively allowing structure stowage without global truss rotations. The SELF method requires high element slenderness ratios, and therefore is limited to very gossamer applications.

Additionally, a survey of potential MEDLS applications was performed and a MEDLS application table developed that includes a description of the MEDLS structural application, design-driving structural and material requirements, the optimal degree of structural hierarchy introduced, packaging requirements and capabilities, manufacturing requirements, and an assessment of the design maturity (TRL). The results of the analytical evaluations of the MEDLS elements to identify member sizes, material strain limits, and stiffness/strength degradation allowed an assessment of the viability and performance of the various concepts, and led to the down-selection of two candidate MEDLS space deployable structures for which conceptual design drawings were generated: a large linear truss structure capable of deploying membrane optics/reflectors; and a large parabolic dish antenna that utilizes a second-order MEDLS support backing structure.

In this Phase I study, MEDLS have been analytically shown to have great potential to provide weight-optimal deployable structures for gossamer space applications, providing the space community with support structures that are as lightweight as the membranes, meshes and wires they are intended to deploy and support.

1.1 SUMMARY OF TASKS PERFORMED AND STUDY OBJECTIVES

During this Phase 1 SBIR study, the following tasks were performed:

- A survey of potential MEDLS applications was made that resulted in the generation of a MEDLS applications table that included: driving structural and material requirements, the optimal degree of structural hierarchy introduced, packaging requirements and capabilities, manufacturing requirements and an assessment of the design maturity (TRL).
- Several MEDLS design tools were developed: The effective Continuum Beam Equations Tool, and A MEDLS Structural Optimization Tool. These tools were used to determine the characteristics of an optimized MEDLS truss element of a given architecture, and to assist in finding the requirements (applications) that MEDLS are most suitable for.
- An assessment of structural hierarchy and its impact on optimal MEDLS structures was performed.
- A hierarchical truss packaging study was performed that included development of global/local elastic collapse mechanisms for

various MEDLS truss architectures using non-linear finite element analyses. The effect of initial member curvature and imperfections was also investigated.

- A unique MEDLS packaging architecture, the Single Elastic Loop Folding (SELF) method was developed. SELF is a simple packaging architecture that works for all lattice structures, as it does not rely on a choreographed sequence of simultaneous maneuvers among the truss nodes, and is customizable to many classes of MEDLS structures.

- From the survey of MEDLS conceptual applications and the results of the MEDLS analyses, two candidate designs were down-selected for further conceptual development: A large linear truss structure capable of deploying membrane optics/reflectors; and a large parabolic dish antenna that utilizes a second-order MEDLS support backing structure. Preliminary design drawings were generated for the two configurations.

The Phase I program objectives are shown in Table 1-1, below:

Table 1-1. Phase 1 SBIR Program Objectives

Objective	Description	Responsible Organization
Applications / Requirements Definition	1. Survey candidate MEDLS applications 2. Define critical design features/performance objectives 3. Define critical requirements 4. Establish initial TRL assessment	ABLE

Configuration Design	1. Define single element MEDLS designs
	  • Basic truss element (bay) architectures
	2. Multiple element MEDLS design
	  • Plate-like (distributed) applications
	  • Beam-like (linear truss) element applications
	  • New packaging architectures
		ABLE

MEDLS System Analysis	1. Define analysis tasks (ABLE/CU)
	2. MEDLS analysis tasks (ABLE/CU)
	  • Single-element strength/stiffness
	  • Develop truss optimization tools to assess design space for MEDLS
	  • Evaluate elastic packaging geometries and collapse modes using non-linear FEA
	  • Evaluate MEDLS element efficiencies / performance relative to material capabilities
	  • Identify critical constitutive metrics / materials
	  • Multiple element strength / stiffness
	  • Strength effects of initial imperfections
		ABLE / CU

Program Management / Reporting	1. Final reporting
	  • Detailed program results summary
	  • Technology development roadmap
	  • Identify recommended Phase II efforts
	  • Generate and submit final Phase I report
		ABLE

1.2   **REPORT ORGANIZATION**

This report is organized as follows:

- ***Technology background, motivation and potential benefits*** – A review of MEDLS technology, it's potential advantages and the motivation for implementation of MEDLS into deployable space structures.

- ***Technology applications*** – A survey of potential applications benefiting from MEDLS technology is presented, including requirements and design criteria used to select the best candidate designs for MEDLS implementation.

- ***Hierarchical truss elements characterization / optimization study*** – development of robust optimization tools used to calculate the dimensions of lightest weight trusses subjected to bending strength and stiffness requirements/constraints.

- ***Hierarchical truss packaging study*** – Non-linear analyses performed to assess global and local elastic collapse mechanisms for several candidate deployed truss architectures. Description of various collapse modes and how they are applicable to stowed packaging. Description and advantages of SELF packaging method.

- ***Manufacturing considerations*** – A brief review of manufacturing issues and ideas for future manufacturing development are presented.

- ***Study results and conclusions*** – A summary of study findings and conclusions is given in this section.

- ***Recommendations for future work*** – Recommendations for future work, including recommended Phase 2 SBIR efforts are provided.

2.0   TECHNOLOGY BACKGROUND AND MOTIVATION

MEDLS represents a new construction architecture and offers benefits to a broad range of space-based structural applications. Relatively large, lightly loaded (i.e. gossamer) applications are targeted as the best implementation of MEDLS because of the high member slenderness ratios required to stay within material strain limitations associated with elastic deployment. Shell-like applications (e.g., electromagnetic spectrum reflectors, thin film optics, radars, solar concentrators, sunshades, solar sails and deployable decoys) and beam-like (synthetic aperture radar, long baseline interferometry) applications are viewed as the space-based uses where MEDLS offers the most significant immediate benefits to the DoD.

A primary spacecraft support structure requirement is to provide dimensional stability. This requirement is often specified as a minimum deployed frequency requirement for deformations due to transient external loads. Space structures typically have relatively high areal densities so that very stiff structures are required to maintain deployed frequency. However, the new generation of gossamer devices (e.g., thin film reflectors, antennas and solar sails) and distributed micro-sensors (e.g., synthetic aperture radar) change this situation, as the deployed areal mass loading for these applications can be extremely small. To maintain frequency, the required structural stiffness decreases in direct proportion to the decrease in mass. State-of-the-art articulated and inflatable rigidizable structures have been optimized for traditional payload mass loadings and do not efficiently scale down to the low mass loading of gossamer devices (because unrealistically thin wall thickness is required). Nonetheless, traditional structures have been applied (with some weight penalty) to gossamer devices.

The traditional approach to support such gossamer devices is to tension cables or membranes and concentrate these loads into a point or line (e.g., NGST sunshade, Halley Rendezvous, NASA Space Technology-7 Proposed solar sail, L'Garde Team Encounter solar sail; also Zmanya and blanket solar arrays such as Hubble and International Space Station). This approach allows the traditional, stouter deployable structural architectures to be used. However, the performance potential of this approach is limited due to the stiffness reducing and nonlinear beam-column effects of compressive loads introduced as structural members react tension. Tension structures can be mass efficient, but only when compensation for the resulting compression loads comes at a minimal "cost" of added mass (e.g., suspension bridges are efficient because they react their compressive loads through the ground).

New support structure architectures and technologies are needed to enable the benefits promised by the new generation of gossamer missions under consideration. MEDLS can provide an ideal support structure for gossamer applications for two primary reasons. First, the new devices have significantly reduced mass loadings, such that the support structure can be reduced beyond what traditional architectures currently are capable of. MEDLS readily scale to this reduced load level. Second, MEDLS provide distributed support for the devices so that tension structures are not needed. This allows MEDLS to be structurally more mass efficient than tension structures.

MEDLS also offer several secondary benefits in their application to precision reflectors. The hierarchical nature of MEDLS allows for a precision, highly faceted surface (1.57 in facets) with deeper backing structure (157 in deep). This circumvents a problem of tetrahedral-type truss reflectors where increasing surface faceting decreases structure depth. Hierarchical construction is also quite amenable to active surface shape correcting. Global surface errors can be corrected with actuators on the 2nd order lattice scale. Local surface errors can be corrected with actuators placed on the 1st order lattice. Lastly, the extreme structural mass efficiency of MEDLS minimizes ground demonstration gravity effects. Large MEDLS structures will be ground demonstrable with simple ground support equipment (such as Helium balloons) placed at a minimum of locations.

2.1   MEDLS DESCRIPTION

Monolithic Elastically Deployable Lattice Structures (MEDLS) have the potential to offer an order-of-magnitude reduction in structural mass over current deployable structure technologies. The MEDLS architecture accomplishes by exhibiting three unique characteristics compared to past efforts in elastically deployable structures:

- They are of monolithic construction in that no hinges or other mechanism-like features are employed. Monolithic structures are amenable to automated manufacturing techniques with a conceivable potential for cost reduction. Such a construction method is also conducive to incorporation of integrated multi-functional elements, such as conductive elements for power, data, antennas and embedded sensor elements. Monolithic structures are also inherently repeatable and stable when deployed (without requiring auxiliary preload), because stability influences due to mechanical joints are avoided.

- The structures are based on beam-like (linear) elements as opposed to more shell-like (distributed) elements. Beam-like elements are chosen over shell structures for the following reasons:

- o Beam elements utilize composite material fiber directional stiffness more efficiently because they can be made from uni-directional (pultruded) composite materials while composite shells generally require a significant portion material fibers oriented in the transverse direction (cross-ply or angle-ply laminates), reducing axial stiffness.
- o Continuous process pultruded composite rods are less expensive to manufacture, as shells generally require time-consuming hand lay-up techniques.
- o Thin, local buckling-limited shell structures are made stronger with reconfiguration of the same material as a truss composed of beam elements.

- The structures do not employ tensioned elements. Tension elements fundamentally reduce the strength of a structure because the tensioning induces compression in some parts of the structure. By eliminating compression elements, the most efficient structure is achieved.

2.1.1 MEDLS Packaging Options

Aside from the fundamental differences outlined above, MEDLS are envisioned as conceptually similar to an ABLE CoilABLE (CoilABLEs employ tensioned diagonals and employ numerous pin joints). As in the CoilABLE, MEDLS subscribe to an engineered packaging scheme that is uniformly repeated throughout the structure. The crux of this effort was to discover efficient packaging schemes allowing completely elastic stowage/deployment; two have been identified as offering the greatest potential for commercialization. The first of these is Single Elastic Loop Folding (SELF), as shown in the sequence in Figure 2.1 and described in detail in Section 4.2. In this method each element forms a single loop as node locations are scaled in one or more directions. Nodes may locally rotate, but the structure does not globally twist. SELF is appropriate for linear and planar structures and all applications, however, it is less efficient than the second method. The second packaging method is twist packaging, Figure 2.2. Twist packaging requires axial rotation of the structure, as the CoilABLE does. This method is highly efficient, however it is most appropriate only for first order linear structures and for many applications the twisting deployment motion is undesirable. Twist packaging is described in detail in Section 4.1.3.
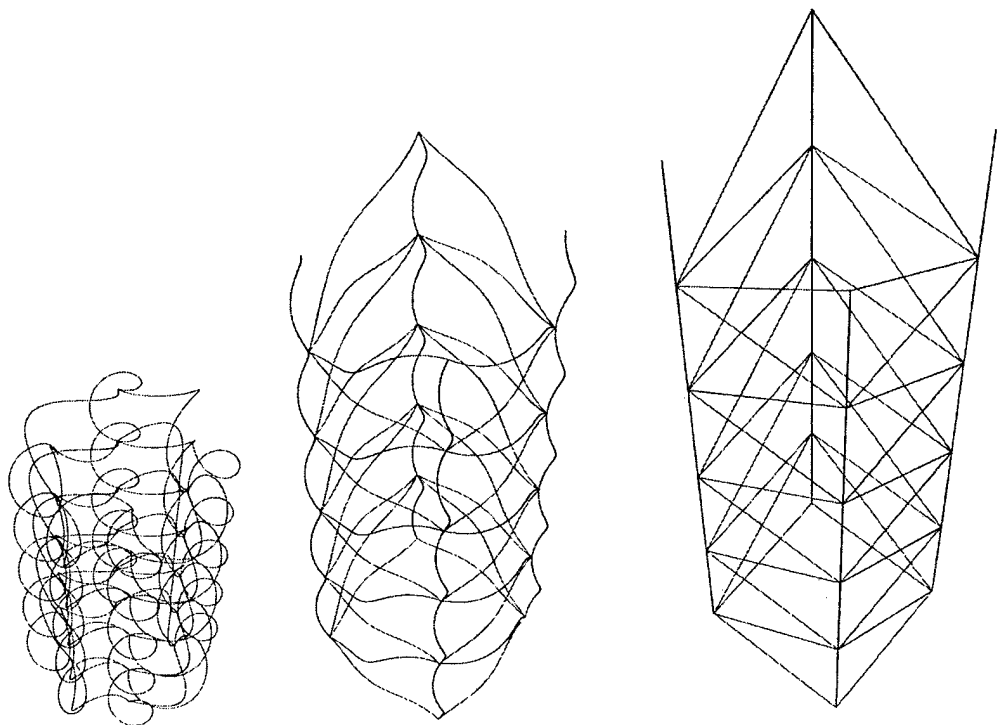
Figure 2-1. SELF Packaging Example

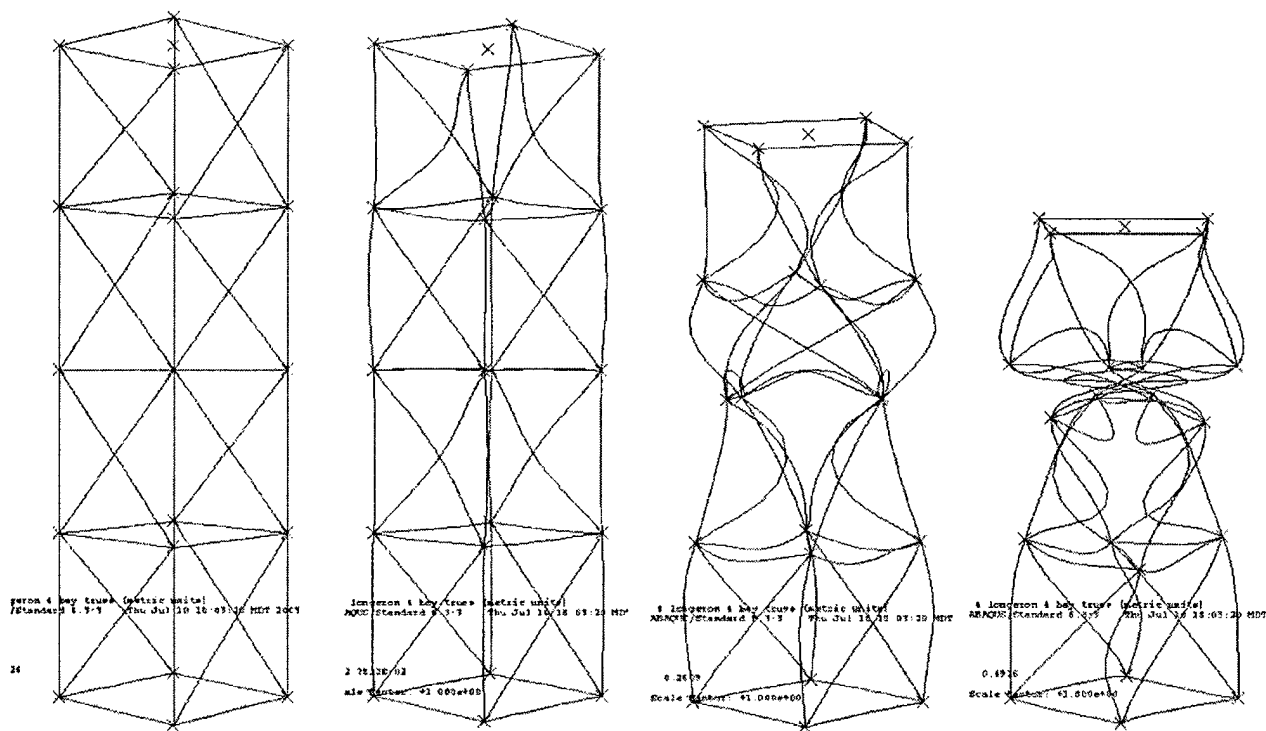

Figure 2-2. Twist Packaging Example

The above packaging architectures require large material strains. In fact, the element slenderness that result in the most efficient deployed structures typically require packaging strains larger than the capability of traditional materials. Figure 2.3 shows element slenderness as a function of material strain for a range of materials and packaging architectures. Twist packaging is the least severe since the longerons are bent approximately to the diameter of the deployed truss. A slenderness ratio on the order of 50 is achievable with graphite material systems and less slender elements are possible with higher strain material systems. For semi-circular packaging (conceivably, packaging architectures exist where each element forms a semi-circle) a slenderness ratio on the order of 157 is achievable with graphite/epoxy and for SELF (each element forms a circle), a slenderness ratio of 314 is required. Clearly, very high strains are required for slenderness ratios in the range of traditional structures, L/d~30. Figure 2.3 provokes two general questions that were addressed in this study. First, what is the slenderness of an optimal truss sized to loads representative of space structures? Second, when a minimum slenderness limit (material strain limit) prevents the optimal slenderness from being attained, how much heavier is the slenderness limited structure than the optimal structure? The answers to these questions were found using advanced truss optimization tools and are described in Section 3.1 of this report.
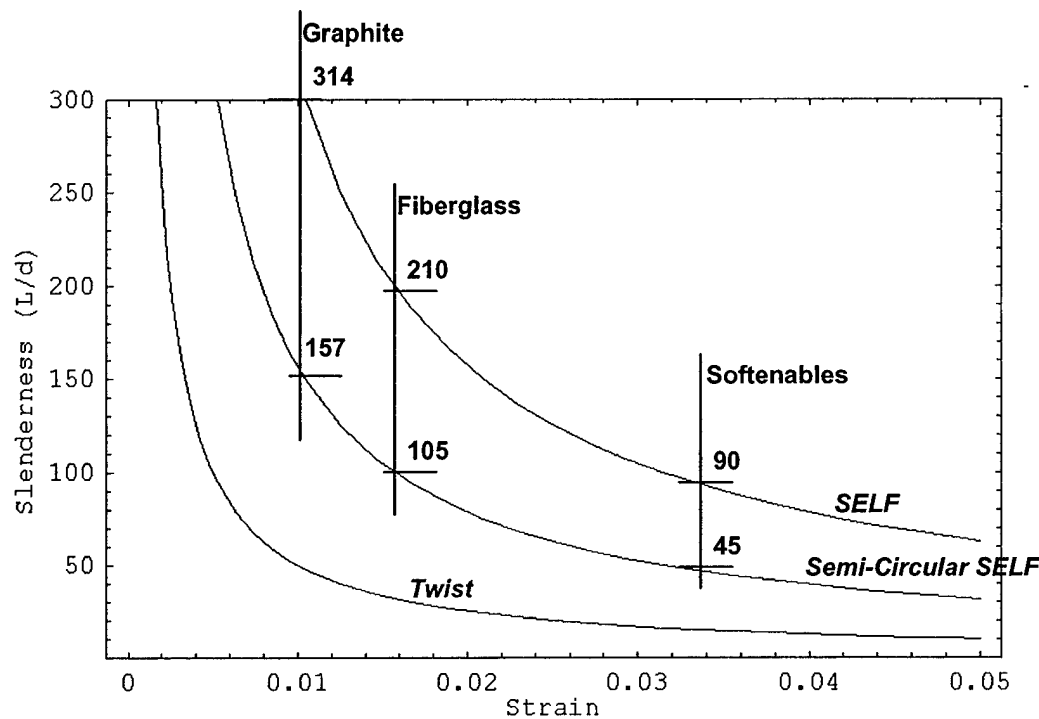
Figure 2-3. Minimum Slenderness Ratio as a Function of Material Strain Limit and Folding Architecture

Before continuing, the definition of an optimal truss structure, as assumed in this study, must be clarified. The required dimensions (element lengths and diameters) of trusses can be calculated purely from a deployed structure requirements perspective, regardless of packaging considerations. The optimized (lightest weight) structure determined in this way is called the optimal truss. Packaging considerations enforce an additional material strain limit such that the optimal slenderness may not be achieved. The best packaging architecture is therefore the one that requires the smallest strain. Design tools developed and analysis results obtained during this Phase I study to determine the characteristics of an optimized truss of a given architecture are described in Section 3 of this report.

2.2   POTENTIAL MEDLS TECHNOLOGY APPLICATIONS

In order to survey possible applications benefiting from MEDLS technology, a "brainstorming" meeting was held at ABLE to solicit MEDLS conceptual ideas and recommendations for requirements from ABLE engineering staff members with expertise in deployable space structures and materials. With the inputs generated from the meeting, an extensive

list of potential MEDLS applications was generated. The conceptual ideas were then categorized based on size, general structural configuration (2D plate, 3D volumetric, etc.), MEDLS element type/behavior, and degree of required packagability. The list of identified MEDLS applications generated, along with additional notes from the meeting are given in Appendix A of this report.

A MEDLS application table was initially developed that was updated as analytical work on the program progressed. The table includes a description of the MEDLS structural application, the driving structural and material requirements, the optimal degree of structural hierarchy introduced, packaging requirements and capabilities, manufacturing requirements, and an assessment of the design maturity (TRL). The results of the analytical evaluations of the MEDLS elements to identify member sizes, material strain limits, and stiffness/strength considerations was used to fill in the table and allow an assessment of the viability and performance of the various concepts. The MEDLS applications are summarized in Table 2.1.

Table 2-1. Potential MEDLS Applications

Application Description	Driving Structural Requirements	Driving Material Requirements	Maximum Degree of Structural Hierarchy	Packaging / Deployment Requirements	Design Maturity (TRL Assessment)
Perimeter stiffening structure for large, parabolic reflector (see HSC spring-back antenna design)	• Circular deployed geometry • Distributed stiffness • Depth to section, providing hoop and torsional stiffness • Good stability	• High specific modulus • Good elastic strain capability • Low CTE	2nd order	• Elastic, linear deployment in two directions • Controlled deployment rate • Low stowed volume	TRL-2: Shown analytically potential improvement in structural mass efficiency over state-of-art
Backside stiffening (shell-like platform) for thin-film tensioned flat or parabolic reflector	• Distributed stiffness • Depth to section, providing hoop and torsional stiffness • Good stability • Good repeatability	• High specific modulus • Good elastic strain capability • Low CTE	2nd order	• Elastic, linear deployment in two (or three) directions • Sufficient deployment force to deploy blanket • Controlled deployment rate • Low stowed volume	TRL-2: Shown potential improvement in structural mass efficiency over state-of-art
Higher-order "CoilABLE" mast for SAR antenna or instrument deployment with longeron and batten components made up of iso-grid type tubes or MEDLS elements that stow axially and elastically	• When deployed, good linear beam stiffness (High EI, GJ and GA) • Strength of structural members (local buckling) • Material in longerons distributed to maximize axial stiffness	• High specific modulus • High degree of directional stiffness (uni-directional composite) • Good elastic strain capability • Low CTE	2nd order	• Rotation of nodes during stowage / deployment preferred • Elastic, linear deployment in one dimension • Sufficiently small stowed radius to fit inside of canister	TRL-3: Currently being developed / tested by ABLE under IRAD and NASA SBIR
Deployable linear support structure for large gossamer distributed-element SAR or thin-film SAR antenna reflector	• Strength of structural members against on-orbit accelerations (local buckling) • Depth to deployed section, providing bending and torsional stiffness • Good stability • Good repeatability	• High specific modulus • Good elastic strain capability • Low CTE	2nd order	• Elastic, linear deployment in up to two directions • Sufficient deployment force to deploy elements • Controlled deployment rate • Low stowed volume	TRL-2: Shown potential improvement in structural mass efficiency over state-of-art

Application Description	Driving Structural Requirements	Driving Material Requirements	Maximum Degree of Structural Hierarchy	Packaging / Deployment Requirements	Design Maturity (TRL Assessment)
Deployable satellite or other space decoy sub-structure – deploys membrane into rigid shape that simulates satellite signature	• Stable and repeatable deployed geometry • Distributed stiffness • Thermal stability	• Radar-signature compatible • High specific modulus • Good elastic strain capability • Low CTE, high temp capability	2nd order	• Elastic, linear deployment in up to three directions • Sufficient deployment force to deploy • Controlled deployment rate • Very low stowed volume	TRL-1: Need more knowledge of requirements for this application
Standard or thin-film solar array deployment and/or photovoltaic support substrate for small or nano-Sat applications	• Distributed stiffness • Depth of section providing good deployed bending stiffness in "low I" direction	• High modulus • Good elastic strain capability	2nd order	• Utilize stored (stowed) strain E for deployment – sufficient deploy force • Controlled deployment rate • Small stowed volume	TRL-2: Shown analytically potential improvement in structural mass efficiency over state-of-art arrays
Higher-order stem or tubular boom	• When deployed, good linear beam stiffness (High EI, GJ and GA) • Strength of structural members (local buckling) • Material distributed to provide maximum bending I	• High specific modulus • Good elastic strain capability • Low CTE	2nd order	• Capability to "roll up" into cylindrical configuration and deploy into final shape elastically • Sufficient stored strain E for deployment motive force	TRL-2: Shown potential improvement in structural mass efficiency over state-of-art booms
Pantographic deployment of panels using distributed "edge links"	• High axial stiffness of link members • Depth of section providing good deployed bending stiffness in "low I" direction	• High specific modulus • Large strain capability	2nd order	• High available strain energy for deployment • Must be able to fold flat • Controlled deployment rate	TRL-1 Concept applied to existing non-MEDLS design
Deployable thermal radiator or sun shield support structure	• Distributed stiffness • Depth to section, providing hoop and torsional stiffness • Good stability • Reasonable	• High specific modulus • Good elastic strain capability • Low CTE	2nd order	• Elastic, linear deployment in two directions • Sufficient deployment force to deploy blanket	TRL-2: Shown potential improvement in structural mass efficiency over state-of-art or current

Application Description	Driving Structural Requirements	Driving Material Requirements	Maximum Degree of Structural Hierarchy	Packaging / Deployment Requirements	Design Maturity (TRL Assessment)
	strength to support radiator / sun shield elements			• Controlled deployment rate • Low stowed volume	concepts
High altitude balloon structures	• Very lightweight • In-plane (hoop) strength	• High specific strength • High strain-to-failure	2nd order	• Sealed to contain gas pressure	TRL-1: Need more knowledge of requirements for this application
Hybrid applications utilizing elastic rigidizable composite or shape-memory alloy materials to increase packaging efficiency	• Very high deployed stiffness to stowed volume ratio	• Very high strain capability in non-rigidized state • High stiffness	2nd order	• Needs heat energy to change phase and actuate / rigidize	TRL-2-3: Research on related structures using these materials being done at ABLE
Elastic MEDLS actuator – uses elastic strain energy of metallic MEDLS structure or foam to produce linear motion / force	• Very high elastic stiffness / unit volume	• Linear stress-strain behavior • No creep	Nth order (foam-like material possible)	• Must be housed in containment housing that can maintain preload on material	TRL-1: Need better definition of concept requirements and potential materials

In order to meet several of the structural and packaging requirements consistently identified during the generation of the MEDLS applications table, and to increase the TRL of the various designs, further detailed development of MEDLS material application and deployment processes will be necessary to investigate the practical design issues associated with implementation of MEDLS.

Some of these key requirements are:

• <u>Deployed strength</u>: Given currently available materials, the ideal configuration for a large deployable space structure must satisfy competing requirements: maximize deployed stiffness and strength, and minimize stowed volume. The former calls for use of high-modulus graphite fibers. Unfortunately there is a nearly inverse relationship between fiber stiffness and strength, so strain capability drops off rapidly as modulus increases. Because of this material strain limitation, MEDLS component members must be very slender to maintain structural margins for elastic stowage. This comes at the

price of low deployed local buckling strengths of those members. The introduction of higher-order latticing methods characteristic of MEDLS can reduce individual element size while increasing local strength by mutual bracing, greatly reducing the strain imposed by stowage, enabling the use of high-performance (and low ultimate strain) graphite fibers. Additional design studies of the implementation of higher-order latticing into the various applications needs be performed.

- <u>Deployment motive force:</u>   Because of the characteristic small member sizing associated with MEDLS, the total amount of strain energy developed when the structure is elastically stowed can be significantly less than that of a typical structure (such as a CoilABLE boom) comprised of larger, lower hierarchy members. In many of the identified applications, the inherent strain energy is used as the primary motive force to deploy the structure and attached membranes, harnesses, instruments or sensors, saving the added weight and complexity of auxiliary deploy-force-generating mechanisms. Further study is required to assess the deploy force margins available for particular MEDLS applications, and whether auxiliary deployment actuation methods are required.

- <u>Controlled deployment rate:</u>   In many applications, when MEDLS structures are deployed, it is desirable to have the rate and sequencing of deployment motion controlled (known) to avoid a rapid dynamic release of strain energy that could break the (slender) MEDLS elements, or cause unwanted dynamic disturbances to a supporting spacecraft. Further design development of ways to simply control deployment rate (with passive damping) and geometric sequence without introducing undue complexity needs to be performed for the various applications.

2.2.1   Down-selected MEDLS Applications for Further Development

A review of the viability of the MEDLS applications identified was performed that included assessment of member sizing, available truss geometries and deployment scenarios developed during the hierarchical truss characterization/optimization and packaging studies. From this review, two candidate MEDLS designs were selected for further conceptual development:

- A large linear truss structure capable of deploying membrane optics/reflectors

- A large flat or parabolic dish membrane antenna/reflector that utilizes a second-order MEDLS support backing structure

Both of the selected applications for implementation of MEDLS architecture are large, gossamer space structures that deploy lightweight and thin membrane payloads; thereby taking full advantage of MEDLS capability for good packaging efficiency, low deployed mass and distributed support, without tensioned members. One beam-like deployed geometry (the linear truss structure), and one shell-like structure (the dish antenna support structure) was selected.

A conceptual drawing of the large linear truss structure is shown in Figure 2.4. Additionally, full deployment animations were generated to better visualize deployment kinematics. The structure utilizes the Single Elastic Loop Folding (SELF) packaging architecture described in Section 4.2. This preliminary MEDLS concept does not incorporate member structural hierarchy beyond 1st –order.
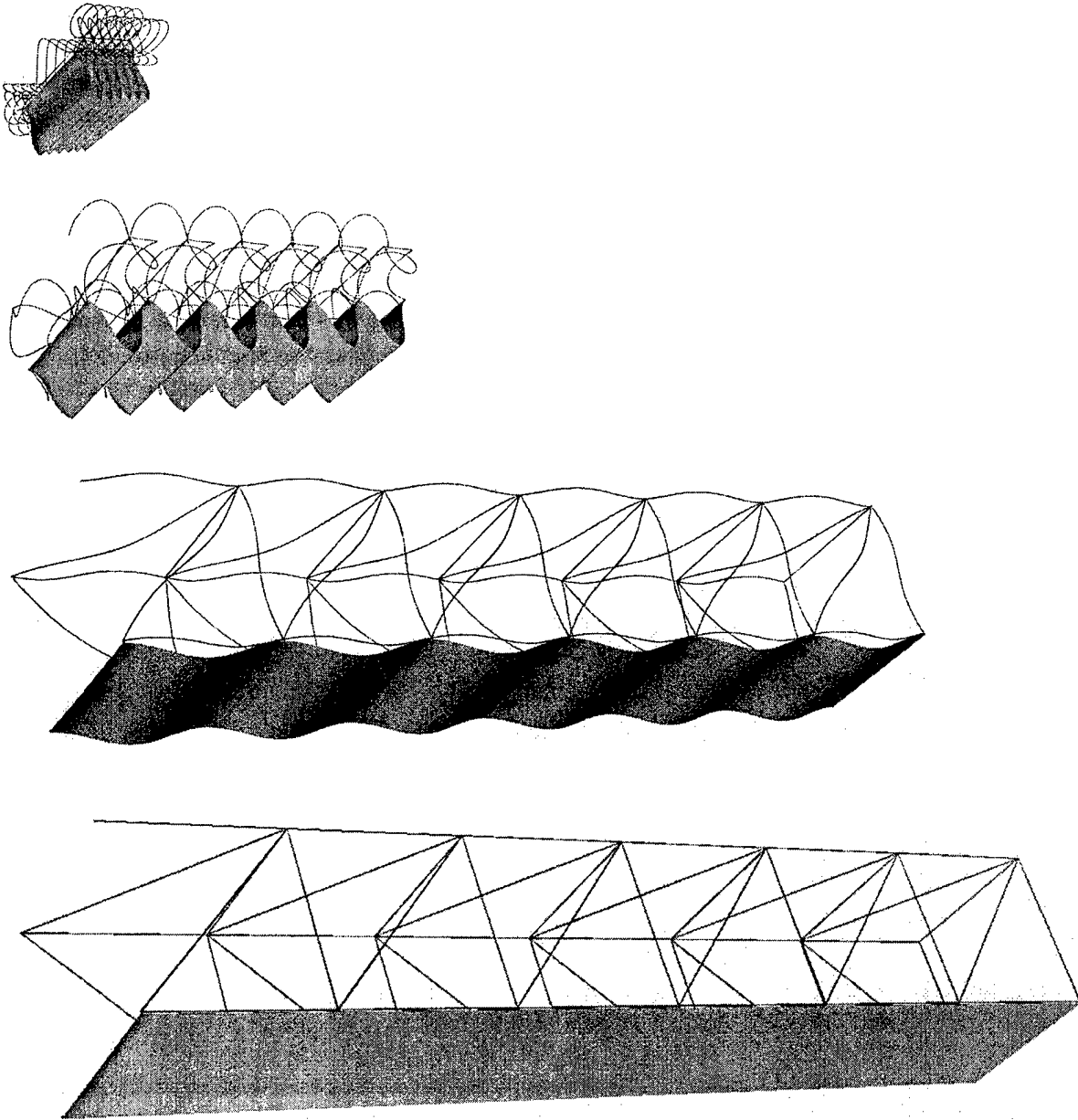
Figure 2-4. LARGE LINEAR TRUSS STRUCTURE DEPLOYING MEMBRANE OPTICS

A conceptual drawing of the large parabolic dish membrane antenna/reflector that utilizes a MEDLS support backing structure is shown in Figure 2.5. This structure implements second-order hierarchy, as shown in Figure 2.5.

Unclassified

Detail of second-order latticing
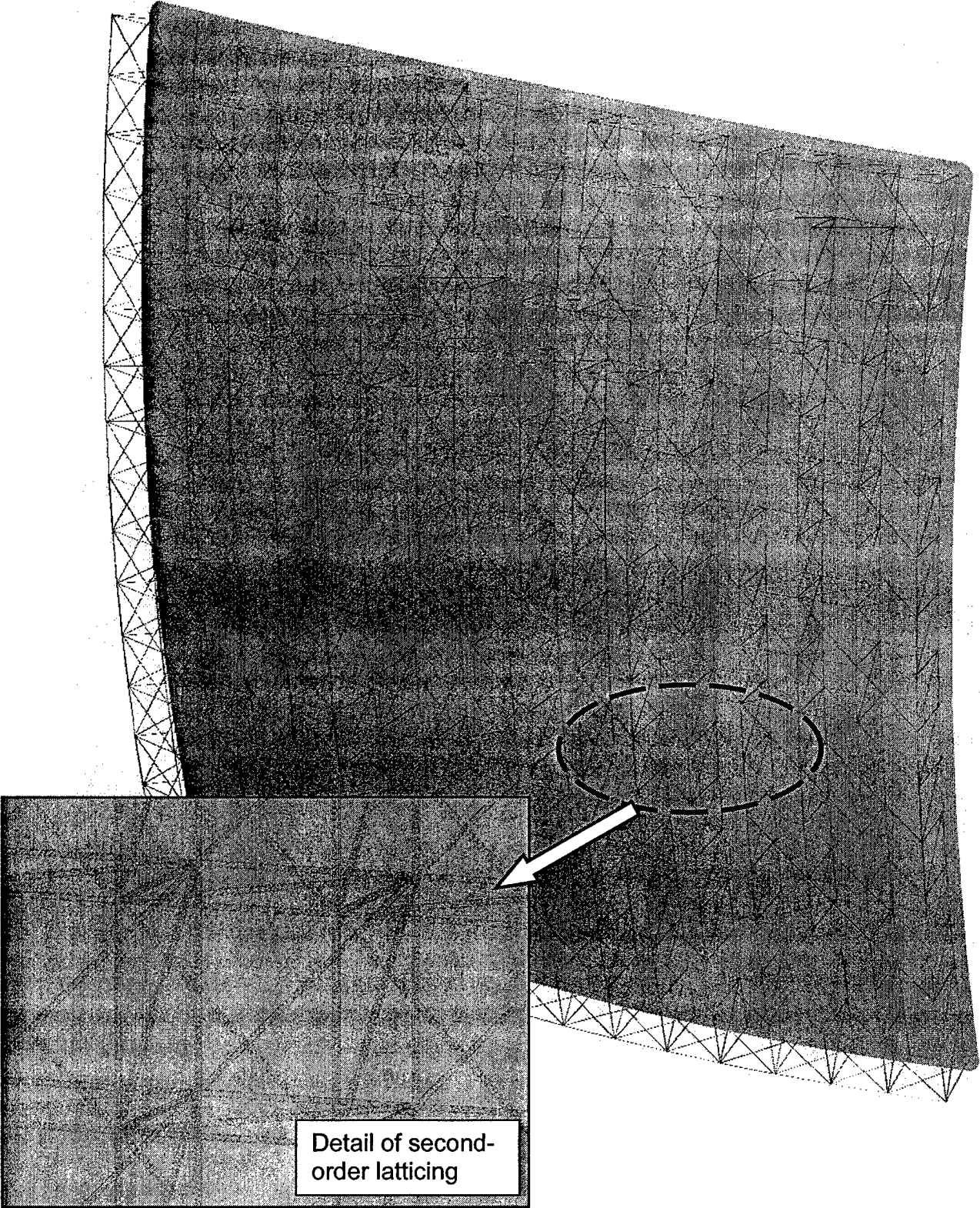
Figure 2-5. Large MEDLS Parabolic Dish Membrane Antenna Support Structure

**3.0   HIERARCHICAL   TRUSS   CHARACTERIZATION / OPTIMIZATION
         STUDY**

During the hierarchical truss characterization / optimization study phase,
the following two MEDLS design tools were developed:

1) Effective Continuum Beam Equations Tool - The direct stiffness
   method of matrix structural analysis was implemented symbolically in
   the Mathematica software program.   The tool is used to generate
   symbolic equations for the deflections and loads in arbitrary three
   dimensional pin jointed truss structures. For MEDLS, the tool is used
   to model a single module (bay) of a much longer truss structure. The
   resulting symbolic equations from a single module are used to derive
   symbolic equations for the effective behavior of a beam constructed
   from many modules.   The tool has been applied to a simple
   three-longeron truss and shown to yield results identical to those
   obtained by Renton (2000) and Noor (1978).   The tool is readily
   adaptable to future MEDLS configurations that may become of interest.

2) MEDLS Structural Optimization Tool - The effective continuum
   equations derived using the previous tool were programmed into a
   nonlinear minimization routine to determine the appropriate dimensions
   for lightest weight first and second order three longeron trusses. The
   tool is based on Microsoft Excel and includes the following features:

   - 1st and 2nd order structure versions.
   - Effective continuum approach for built up truss strength and stiffness
     properties.
   - Stiffness reductions due to initial straightness imperfections.
   - Strength reductions due to initial straightness imperfections.
   - Estimates node (connection point of intersecting elements) mass.
     Node mass is a significant fraction (20-50%) of the total mass and was
     calculated consistently throughout.   The mass of each node is
     assumed to be equal to the mass of a sphere with radius equal to
     twice the longeron diameter.

   The tool is primarily documented in the paper written during the course
   of this Phase I study entitled: "Performance Trends in Hierarchical
   Truss Structures," presented at the 44th AIAA Structures, Dynamics
   and Materials Conference in Norfolk, Virginia (and provided as
   Appendix C to this report). Since its presentation, the tool has been
   updated to include the features listed above. The tool considers a
   beam of arbitrary length subject to both bending stiffness and bending
   strength requirements and finds the lightest weight per length bay that

satisfies the requirements. For the first order system, bay length and diameter are optimized along with longeron, batten and diagonal diameters (5 parameters total). The effective beams are assumed regular in that member diameters do not vary along their length. For the second order system, bay length and diameter are optimized for the first and second order elements (8 parameters) along with (independently) the diameters of the three 1st order elements of the three 2nd order elements (9 parameters) for a total of 17 optimization parameters. This optimization procedure was carried out for a range of bending stiffness and strength requirements. While the tool is easily adaptable to alternate architectures it is currently configured for the 1st and 2nd order three longeron trusses shown in Figure 3.1.
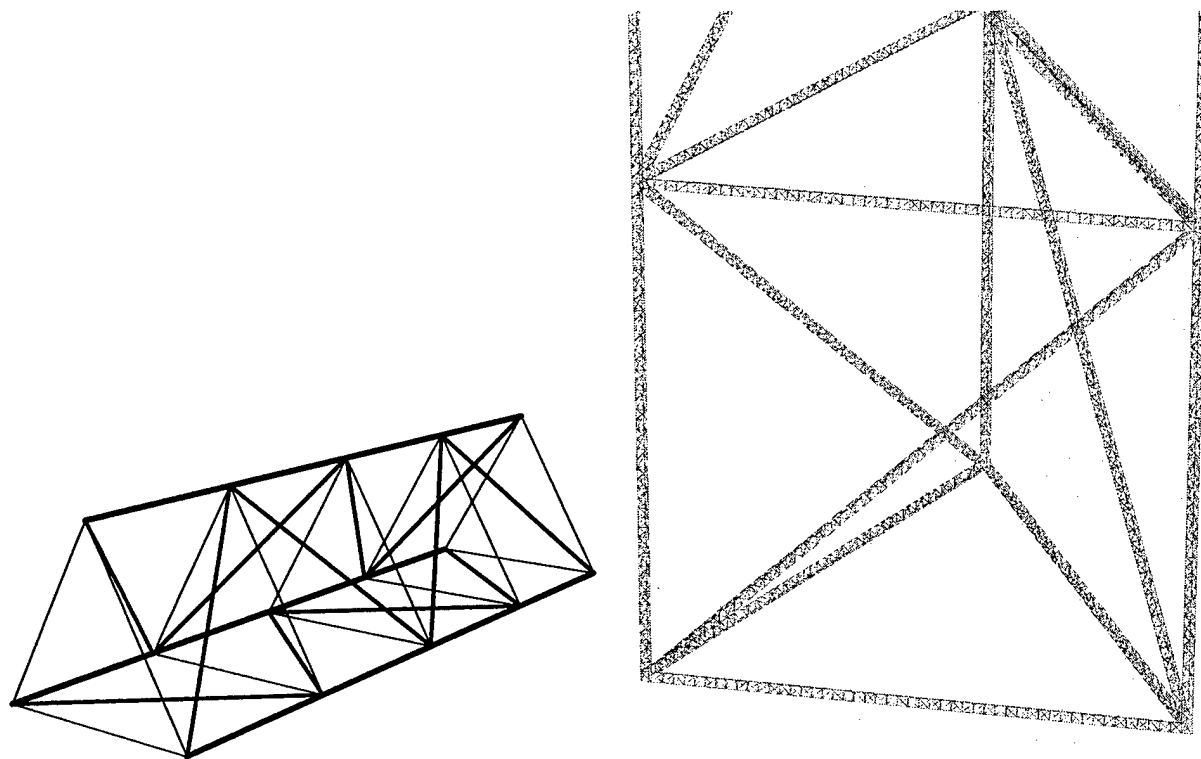
Figure 3-1. 1st and 2nd Order Three Longeron Trusses

3.1   OPTIMIZED TRUSSES

The MEDLS Structural optimization tool was used to determine the characteristics required of optimized trusses. Rather than focusing on a specific application, the design tool was used to find the requirements (applications) that MEDLS are most suitable for. To accomplish this, the truss requirements were reduced to bending stiffness and bending strength requirements and other structural requirements were inferred from these. The plots in Figures 3.2 and 3.3 show the weight per length of

optimal 1st and 2nd order trusses for a range of these two requirements as well as element straightness imperfection, c (see AIAA paper, Appendix C). The assumed truss material properties are E = 200 GPa, r =1660 kg/m^3 and s_{max} = 100 MPa and correspond to a high-strength graphite/epoxy material system. The results in the figures are for optimized lightest-weight structures and represent an ideal to which MEDLS and other structures can rationally be compared.

Unclassified
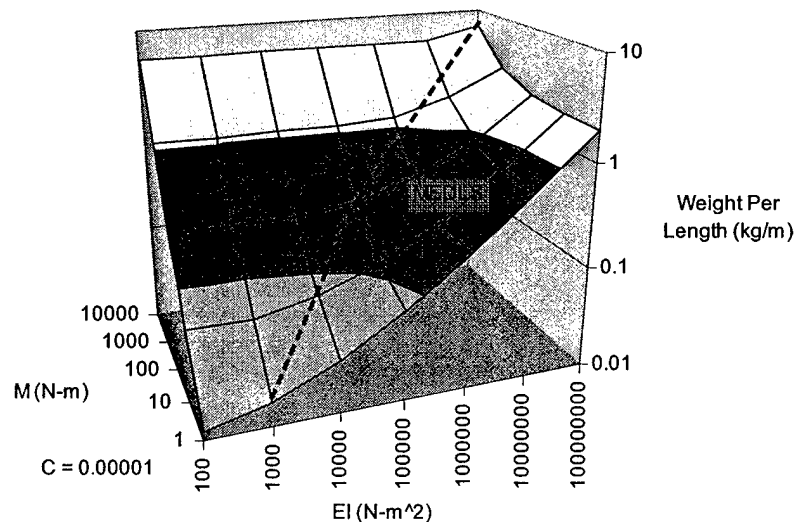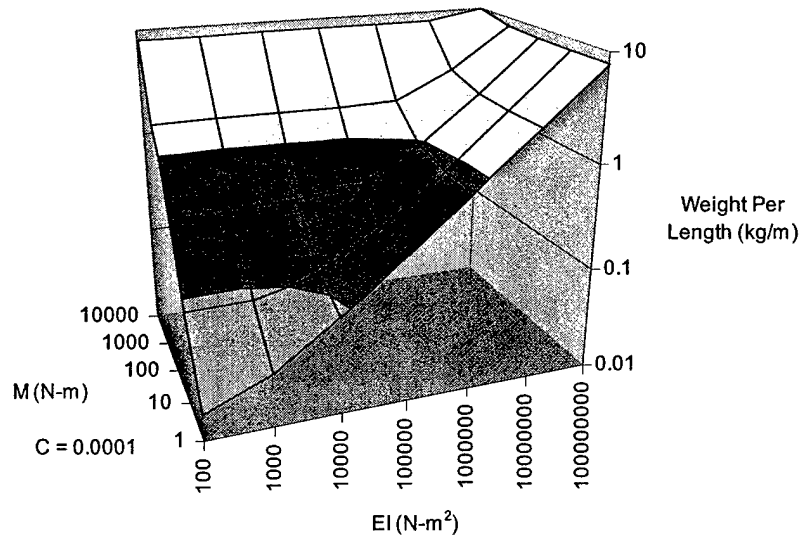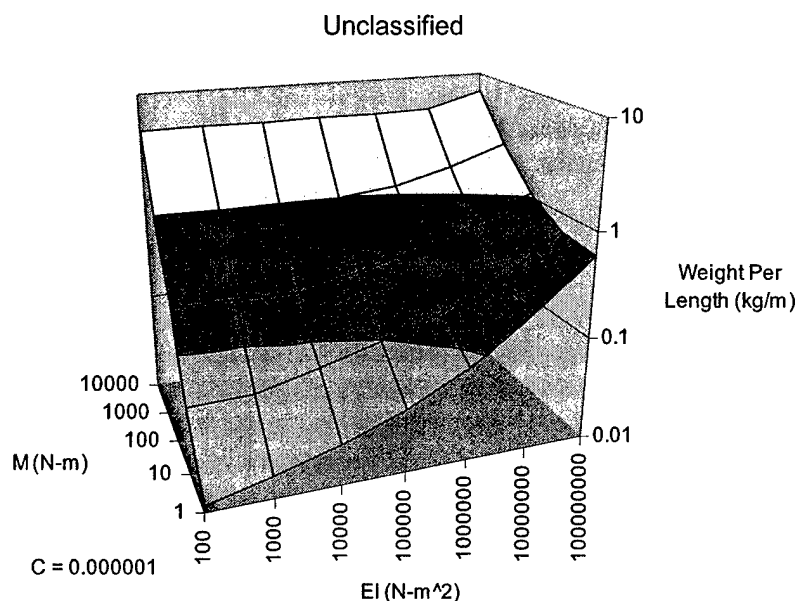
Figure 3-2. Weight Per Length of 1st Order Optimized Trusses Without Slenderness Constraints for c = 0.0001, 0.00001 and 0.000001
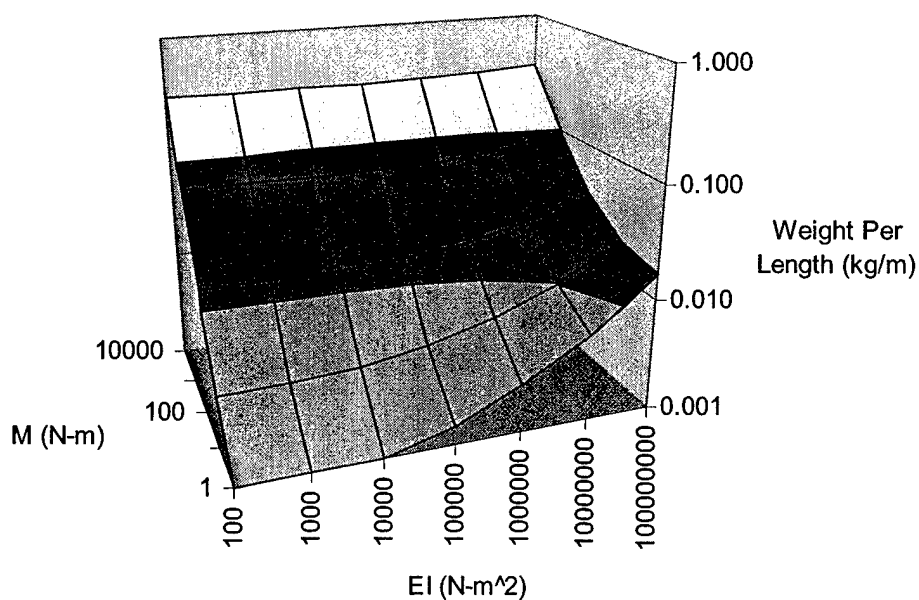
Figure 3-3. Weight Per Length of 2nd Order Optimal (No Slenderness Restriction) Trusses for c = 0.00001 (Reasonable Straightness Imperfection)

Additional constraints due to packaging considerations, namely slenderness, can prevent these optimal dimensions from being attained and increase the truss weight. Figures 3.4 through 3.6 show the effect of

Unclassified

a minimum slenderness restriction on weight efficiency. The ratio of the weight per length of a slenderness-restricted truss to the weight per length of an optimal (no slenderness constraint) truss is plotted. When the weight increase incurred with the minimum slenderness restriction is greater than approximately two to three (shown on the vertical axis), MEDLS are not an efficient architecture – a lighter weight CoilABLE or articulated truss could be made. For some requirements, the optimal structure has a slenderness ratio on the order of 300, implying no weight penalty. MEDLS are ideal for these gossamer applications and the general region for which this occurs is shown in Figure 3.5 for $c = 0.00001$.

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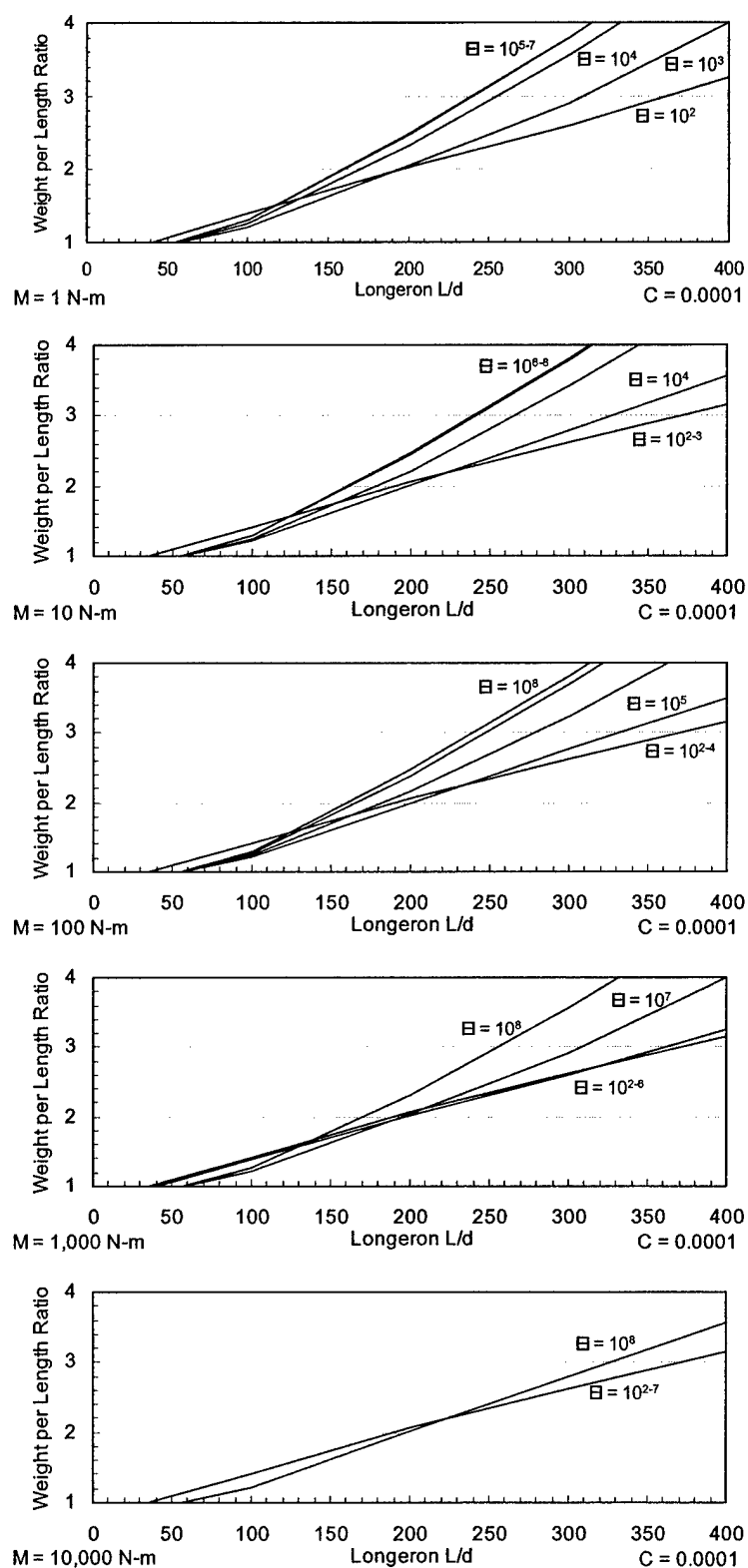
Figure 3-4. **Ratio of Slenderness Restricted Truss Weight Per Length to Optimal Truss Weight Per Length for 1st Order Trusses with c = 0.0001 (Large Imperfections)**

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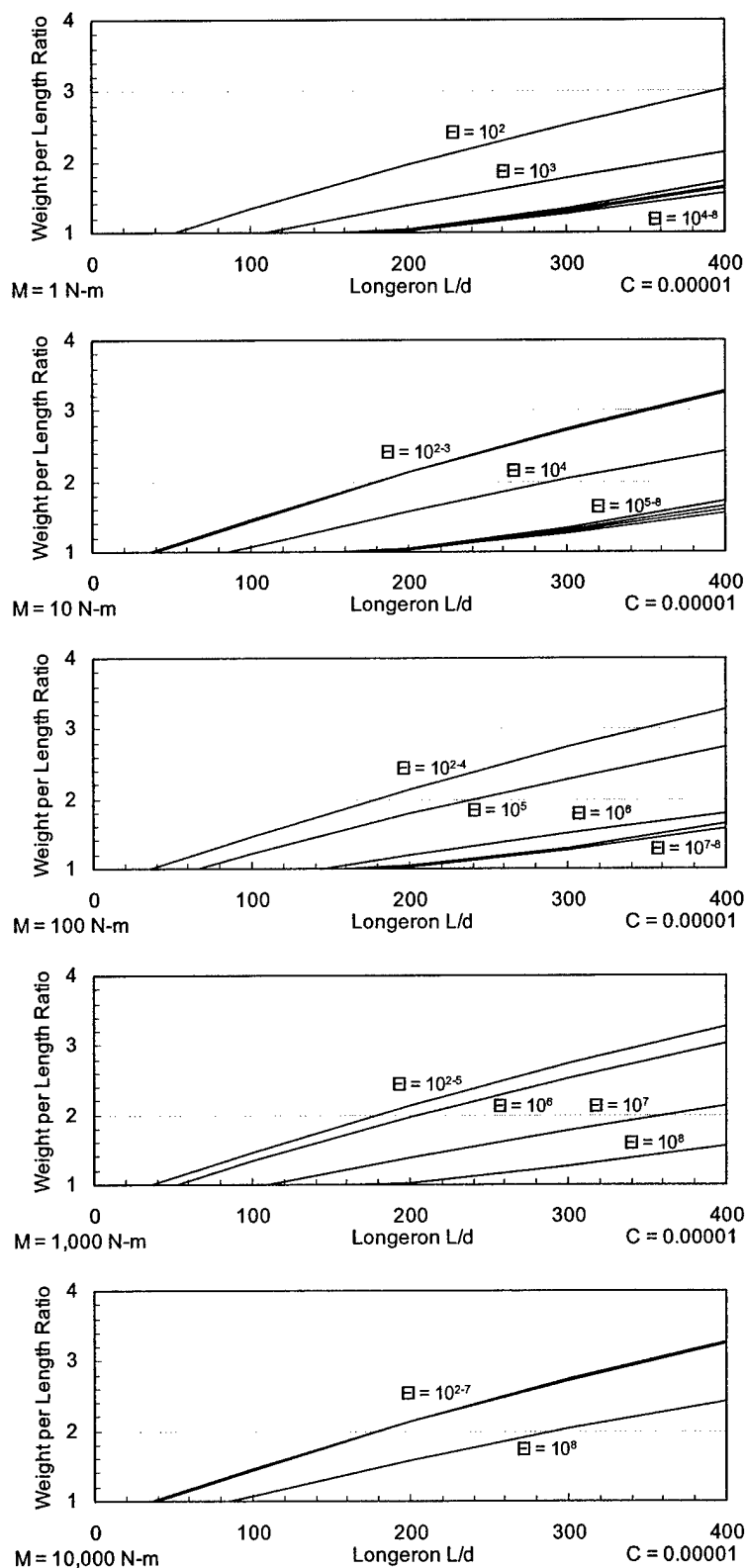
Figure 3-5. Ratio of Slenderness Restricted truss Weight Per Length to Optimal Truss Weight Per Length for 1st Order Trusses with c = 0.00001 (Reasonable Imperfections)
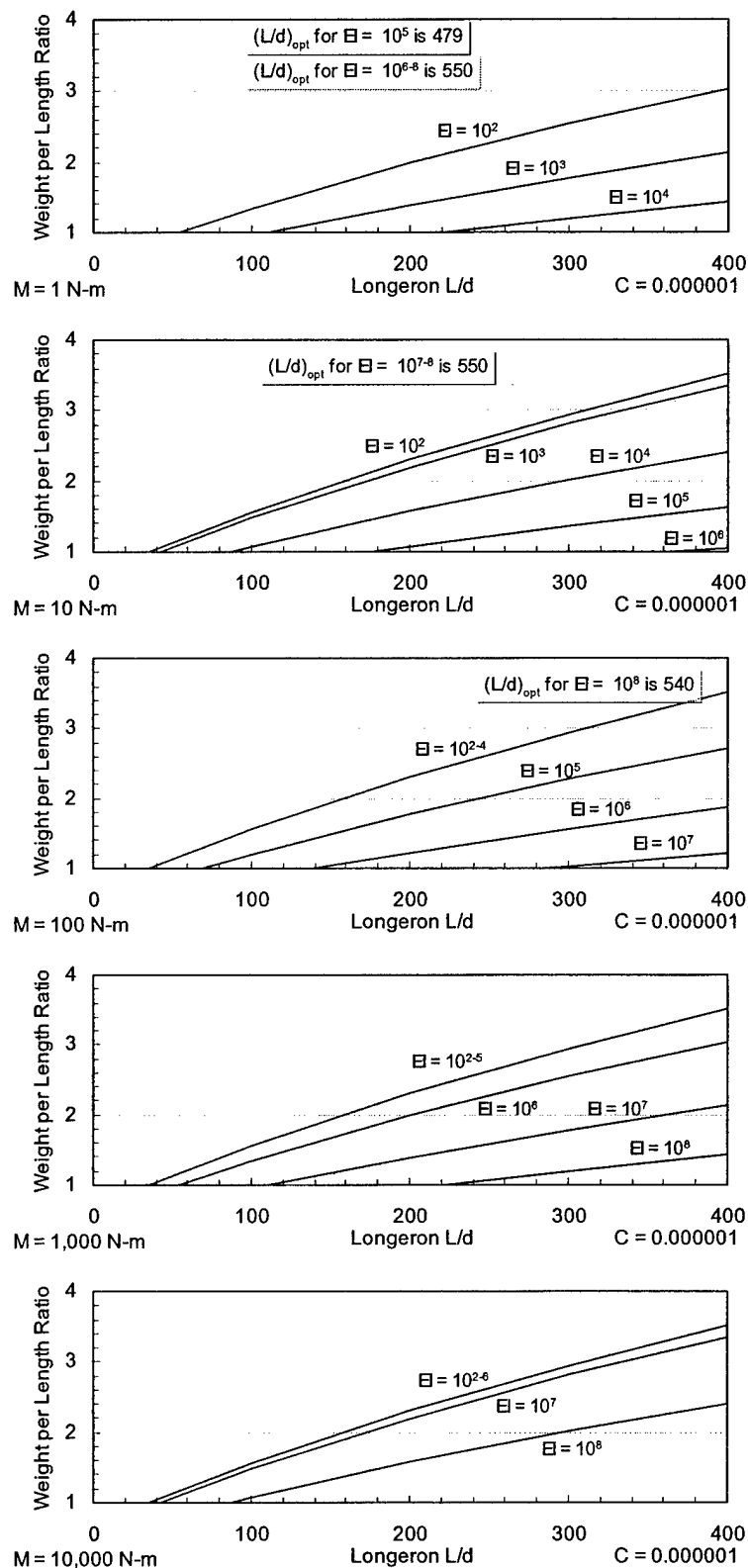
Figure 3-6. Ratio of Slenderness Restricted Truss Weight Per Length to Optimal Truss Weight Per Length for 1st Order Trusses with c = 0.000001 (Small Imperfections)

The plots reveal a very important material requirement: the minimum element straightness required for efficient slender structures. For slendernesses on the order of 300, c<0.00001 is required. Larger imperfections (c=0.0001) result in structures that are 2-4 times heavier. Structures with smaller imperfections (c=0.000001) are nearly the same weight as structures with c=0.00001, which is considered a reasonable imperfection. For reference, c=0.00001 corresponds to an element outer surface fiber strain of ~0.01%.

3.2   1ST VS. 2ND ORDER STRUCTURAL HIERARCHY

This Phase I effort also addressed the issue of structural hierarchy. Increasing structural hierarchy is often associated with more efficient structures and the monolithic nature of MEDLS is highly conducive to unprecedented orders of structural hierarchy. However, as documented in AIAA paper (Appendix C), it was generally found that 2nd order hierarchy is optimal for most reasonable requirements. 3rd order hierarchy is optimal in extremely high stiffness and lightly loaded applications. While there may be applications where 3rd order structures are optimal, they are currently peripheral and there consideration is left for future research. Figure 3.7 shows how the performance of 2nd order optimal trusses compares to 1st order optimal trusses; the 2nd order trusses are 16 to 100 times lighter than the 1st order trusses with the greatest weight reduction occurring for the low strength and high stiffness trusses typical of space applications.
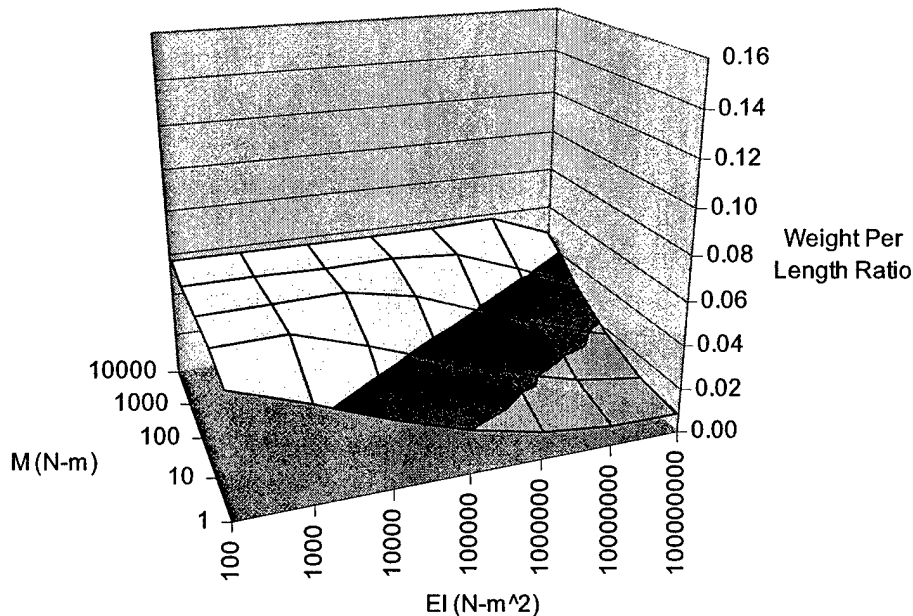
Figure 3-7. Ratio of Optimal 2nd Order Truss Weight Per Length to Optimal 1st Order Truss Weight Per Length with c = 0.00001 (Reasonable Imperfections)

With 2nd order trusses, it is again prudent to question whether or not slenderness restrictions significantly reduce the efficiency of the structure. Such was investigated with the 2nd order truss optimization tool and the results are presented in Figures 3.7 and 3.8. Figure 3.7 shows the ratio of the weight per length of a slenderness restricted (slenderness>300) 2nd order truss to an optimal 1st order truss. The 2nd order-restricted truss is 7 to 100 times lighter than the 1st order optimal truss and the greatest weight savings occur for low strength and high stiffness applications. Figure 3.8 shows the ratio of the weight per length of a 2nd order slenderness restricted (slenderness>300) truss to a 2nd order optimal truss. The slenderness-restricted truss is 1.5 to 2.5 times heavier than the optimal truss, indicating a greater mass penalty for 2nd order trusses than occurs for 1st order trusses with slenderness restrictions. Even so, 2nd order restricted trusses are always much lighter than 1st order trusses and alternative technologies in this load range are non-existent.
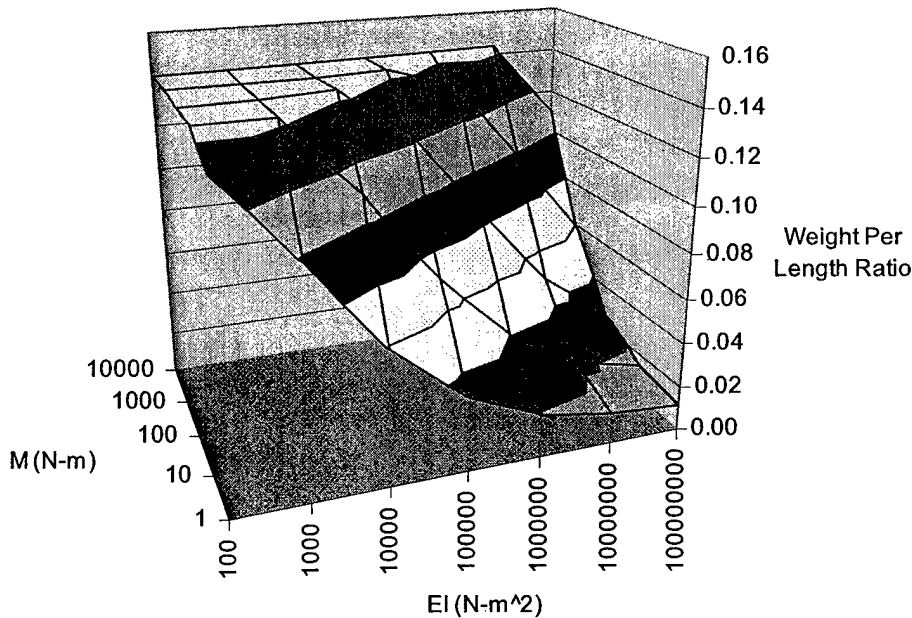
Figure 3-8. Ratio of 2nd Order Slenderness Restricted (Slenderness > 300) Truss Weight Per Length to Optimal 1st Order Truss Weight Per Length with c = 0.00001 (Reasonable Imperfections)

4.0   HIERARCHICAL TRUSS PACKAGING STUDY

Both computational and classical mechanical analyses were applied to the study of candidate elastic packaging architectures for MEDLS structures. The primary objective of these analyses is to predict the strains involved in elastically packaging hierarchical truss elements to allow for the rigorous design of their members. This section describes both the analytical methods which were applied and corresponding results for a number of structural architectures and packaging approaches.

Two general packaging approaches are described in the following subsections. The first, termed Global/Local Elastic Collapse (GLEC), involves packaging MEDLS elements through primarily global loading in an effort to simplify the packaging and controlled deployment process. The more carefully controlled Single Elastic Loop Fold (SELF) approach is then described. In each case, the scalable strains required for packaging are identified and related to required member slenderness.

4.1   GLOBAL/LOCAL ELASTIC COLLAPSE

One intuitive approach to packaging MEDLS class truss structures is through elastic buckling modes. The coupling between global and local elastic buckling in these structures provides the opportunity to package 1st order elements by collapsing them through controlled global loading and displacements. Given the geometric complexity of these buckling modes and post-buckling collapse deformations, a computational approach was selected to study the strains involved in the Global/Local Elastic Coupling (GLEC) approach.

4.1.1   Non-Linear Analysis Methodology

The ABAQUS/Standard finite element analysis package was applied to this modeling effort. Candidate MEDLS architectures were meshed in ABAQUS input scripts that allowed their global and local properties to be parameterized for studying design trends. B31H hybrid beam elements were assembled typically with 10-20 elements per truss member. Member slenderness (L/D) was kept above 100 to ensure bending deformations dominated the packaging process.

Elastic collapse analyses involved a two-step process. Initially, the first few global elastic buckling modes of an architecture, boundary condition, and load case were identified through a linear static Eigen analysis. The resulting Eigen modes were then used as initial geometric imperfections to provide control over the subsequent large deflection collapse analyses. Initial imperfections were typically chosen near L/100.

Both the basic Newton and more advanced Riks analyses were applied to model the strongly nonlinear large deflections involved in elastic collapse. While the Riks analysis provides a more robust integration process in the case of unstable elastic buckling, the classical Newton technique was found to provide accurate and more computationally efficient solutions in the majority of cases. In particular, the use of static Eigen modes as initial imperfections allowed the Newton analyses to traverse the initial unstable displacements involved in these packaging analyses.

As described below, it was found that the maximum member curvature involved in the elastic collapse modes serves as an excellently scalable metric for each packaging case. The product of the maximum member curvature and member length provides a non-dimensional metric that is independent of global and local geometries. The values and interpretation of this metric are presented in the following sections.

4.1.2   Deployed MEDLS Architectures Studied

Three general boom architectures were studied in the GLEC analyses. As illustrated in Figure 4.1, these consisted of: 3-longeron, 4-longeron, and octahedral designs. Single and double bay models were generated for each case and four bay models were developed for the 4-longeron and octahedral designs.
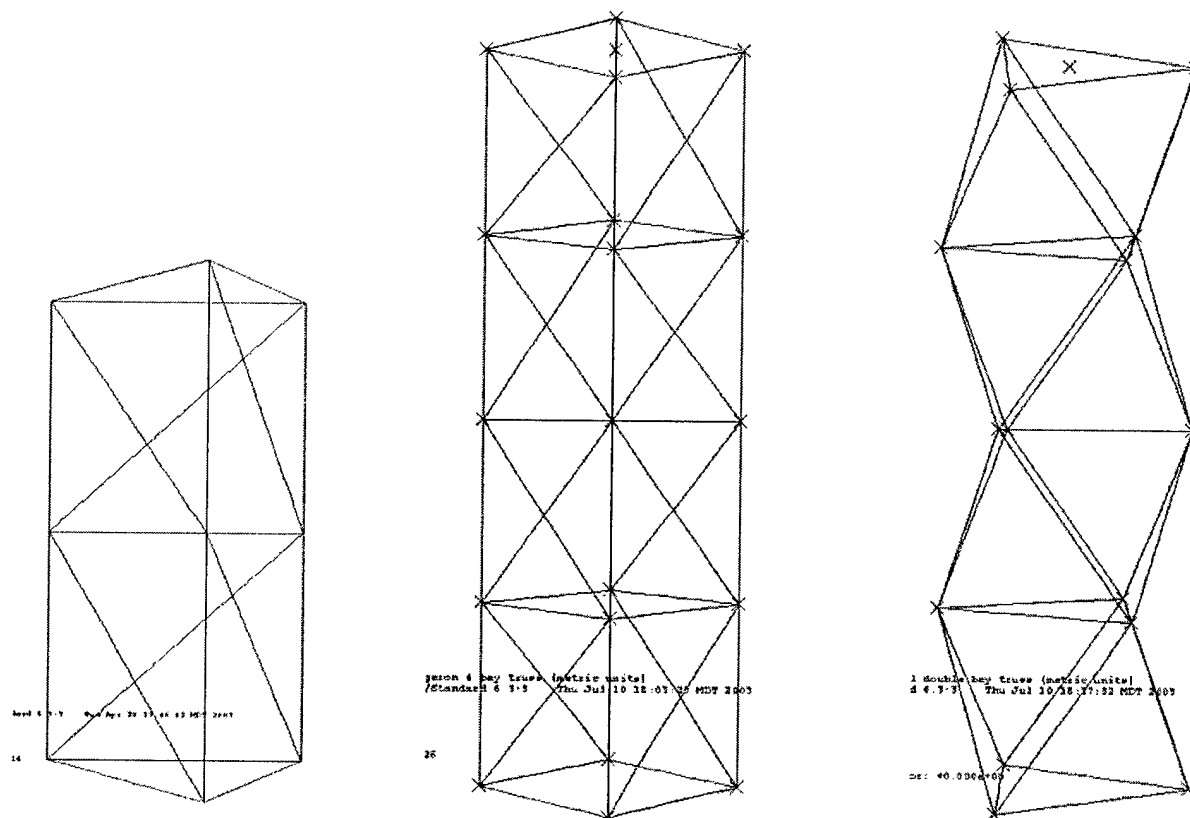
**Figure 4-1. Three General Boom Architectures were Studied
in the GLEC Analyses**

4.1.3   Collapse Modes

In each model, the base nodes of the boom or bay were fixed and the uppermost nodes were displaced axially to induce collapse. The lateral constraints on the uppermost nodes included both fixed and free conditions. Again, the intent of the GLEC packaging approach is to simplify the packaging process by involving primarily global control.

In some cases, the fixed-free conditions allowed relative rotations of the batten planes to develop due to the reactions of the diagonal elements during collapse. An example of this behavior is shown in Figure 4.2. In this case, a 3-longeron single bay model is collapsed through axial displacements of the uppermost nodes in the absence of lateral constraints. The resulting torsional response is most evident in the third frame of this figure.
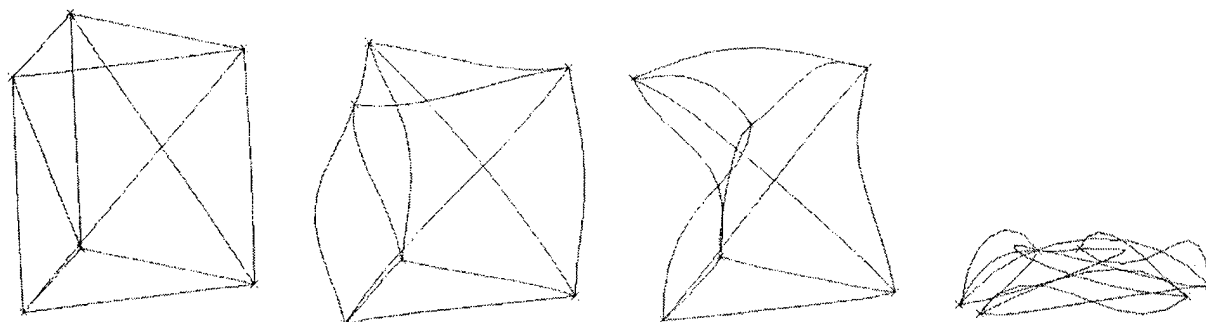
Figure 4-2. Fixed-Free Axial Collapse of 3-Longeron Single Bay Model

Another important trait found in the collapse of multiple bay models is localization of the deformations. A clear example of this is shown in Figure 4.3. In this case, the fixed-free axial collapse of the 4-longeron double bay model results in an initially global collapse mode followed by the complete collapse of the uppermost bay.
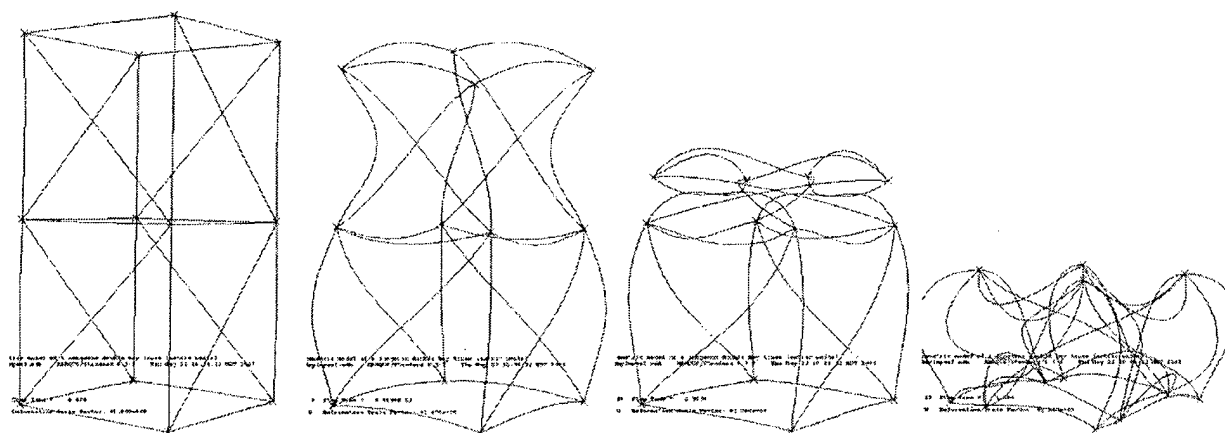
Figure 4-3. Fixed-Free Axial Collapse of 4-Longeron Double Bay Model

This localization is even more pronounced in the four bay model shown in Figure 4.4. Again, the global response appears to cascade from the top bays down. In this fixed-free collapse of the four bay 4-longeron model, we also observe the upper bays being driven inside of the lower bays. If such modes were applied to the packaging process, this may indicate a need to drive collapse at additional internal locations.
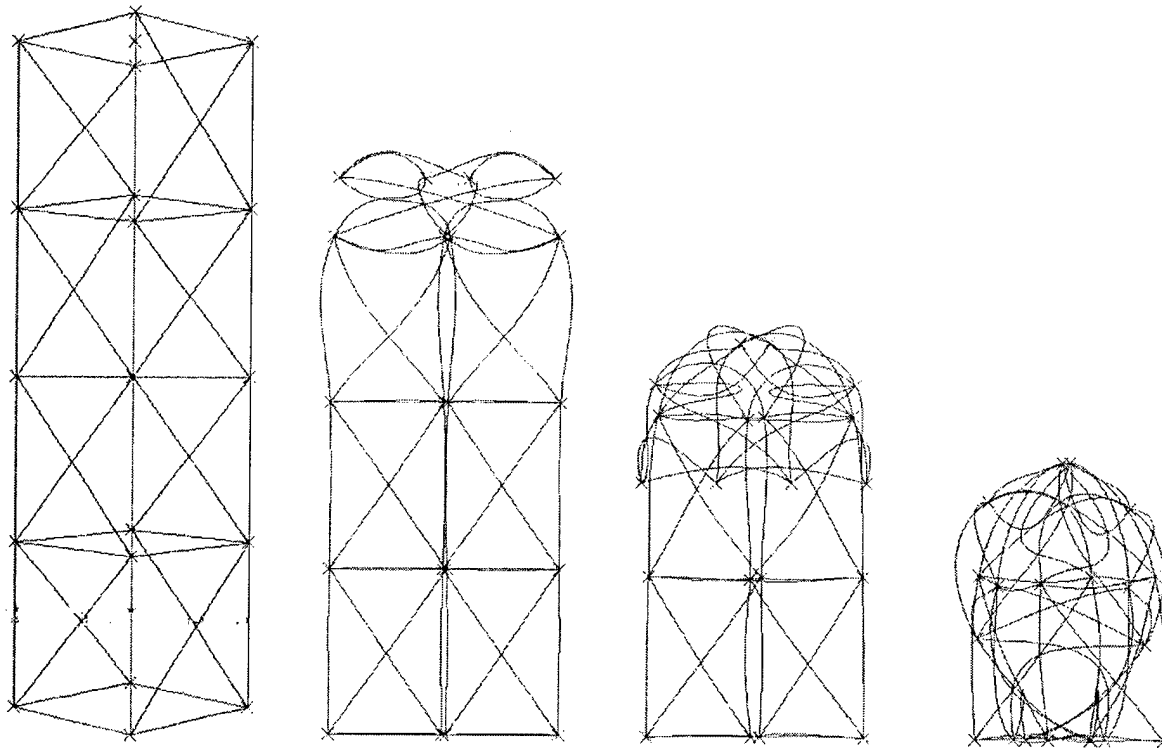
Figure 4-4. Fixed-Free Collapse of 4–Longeron 4-Bay Model

As seen in Figure 4.5, the fixed-sliding end condition initially deformed in a more evenly distributed collapse shape, but quickly localized into a final collapse similar to that of the fixed-free case. This bay-by-bay localization appears to be analogous to a pair of nonlinear softening springs in series – since either spring loses stiffness during compression, any uneven deformation quickly leads one spring to collapse ahead of the other.
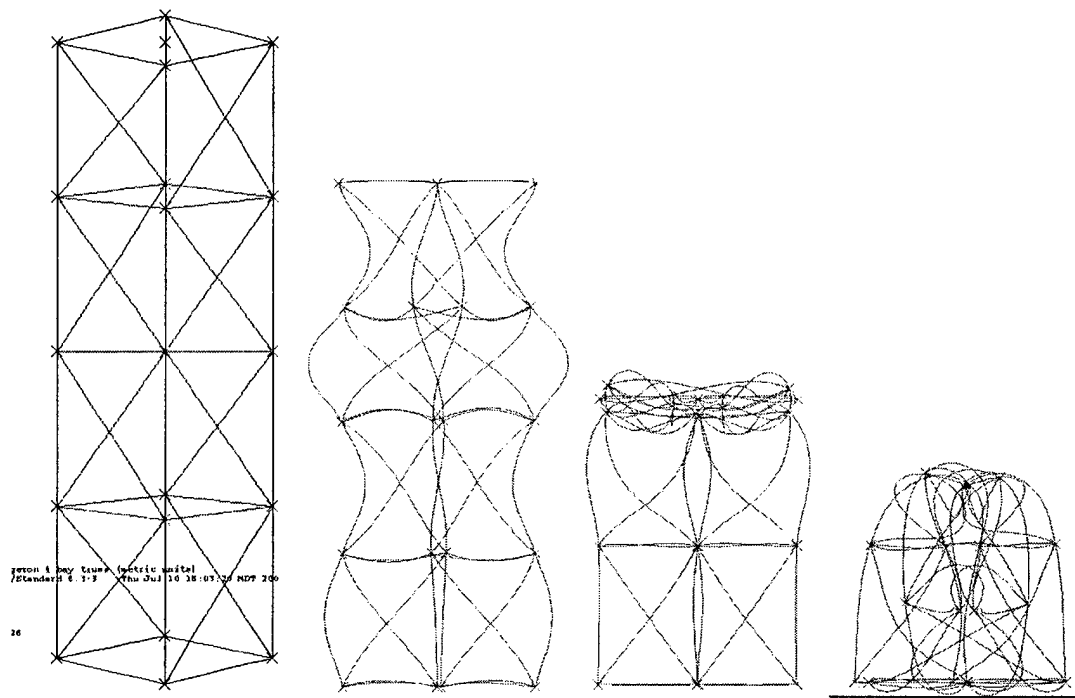
Figure 4-5. Fixed-Sliding Collapse of 4-Longeron Architecture

In an effort to avoid this localization in the collapse process, torsional displacements of the uppermost bay were added to the model cases. An example of such an analysis is shown in Figure 4.6. Again, localization of the collapse is observed. In this case, the upper bay does not collapse as readily due to the additional constraints involved in applying the torsional displacements.

It is useful to consider the potential differences between this modeling case and the packaging of a CoilABLE longeron boom. At least two architectural differences may contribute to the localization seen in the GLEC analyses. Initially, the relative truss member properties are much different in the case of CoilABLE longeron designs. Specifically, the tensioned diagonals and sprung battens of a CoilABLE longeron truss are not well represented by the uniform member properties in the current GLEC models. The resulting collapse mode in a CoilABLE longeron is focused on decoupled global bending of each longeron, which is not possible in the case of the MEDLS assemblies.
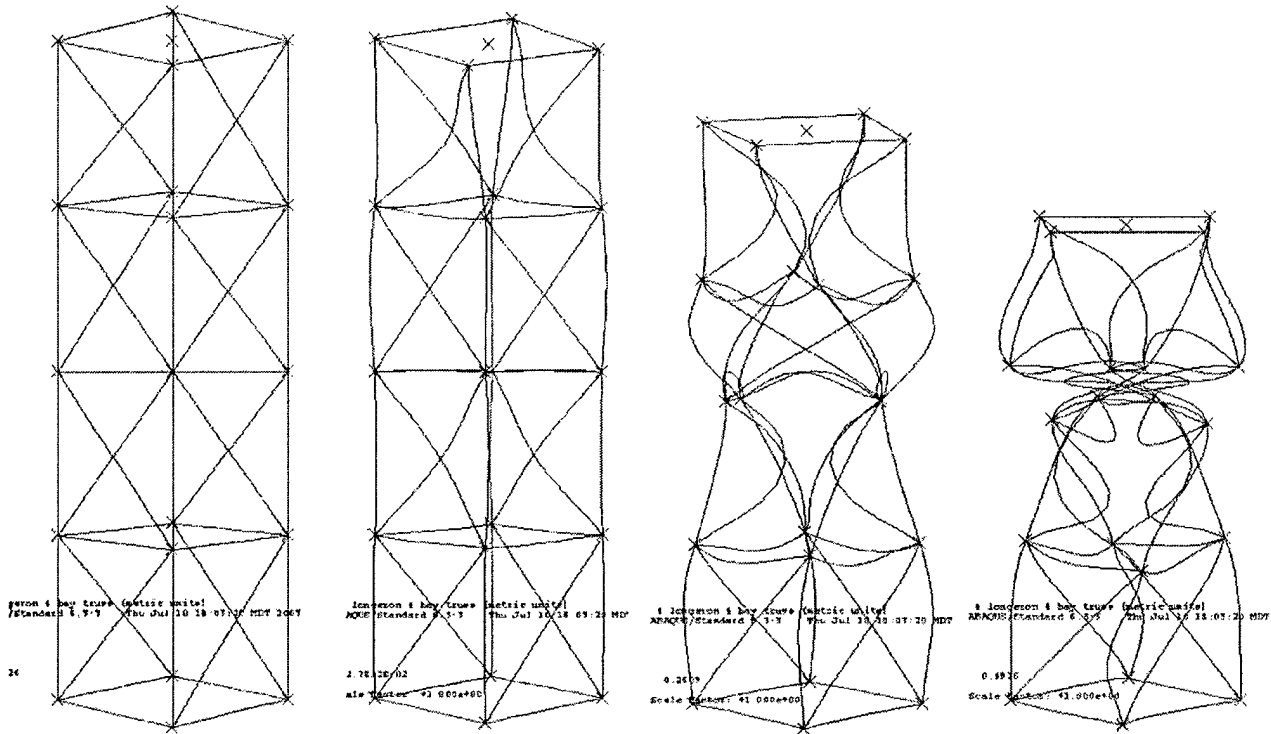
Figure 4-6. Torsional Collapse of 4-Longeron Truss Architecture

The octahedral truss architecture was of particular interest due to its axial symmetry. In principle, it was expected that the equivalence of this architectures longeron and diagonal elements might lead to relatively uniform collapse behavior. However, single and double bay models of the octahedral design quickly revealed that this symmetry is not necessarily reflected in its large displacement collapse modes.

Figure 4.7 illustrates the collapse of the four bay octahedral model under fixed-free conditions. Again, the localization of collapse to the uppermost bays was observed. Figures 4.8 and 4.9 provide a similar result for the fixed-sliding and torsional cases, respectively. The octahedral design was found to more evenly distribute packaging strains, however.
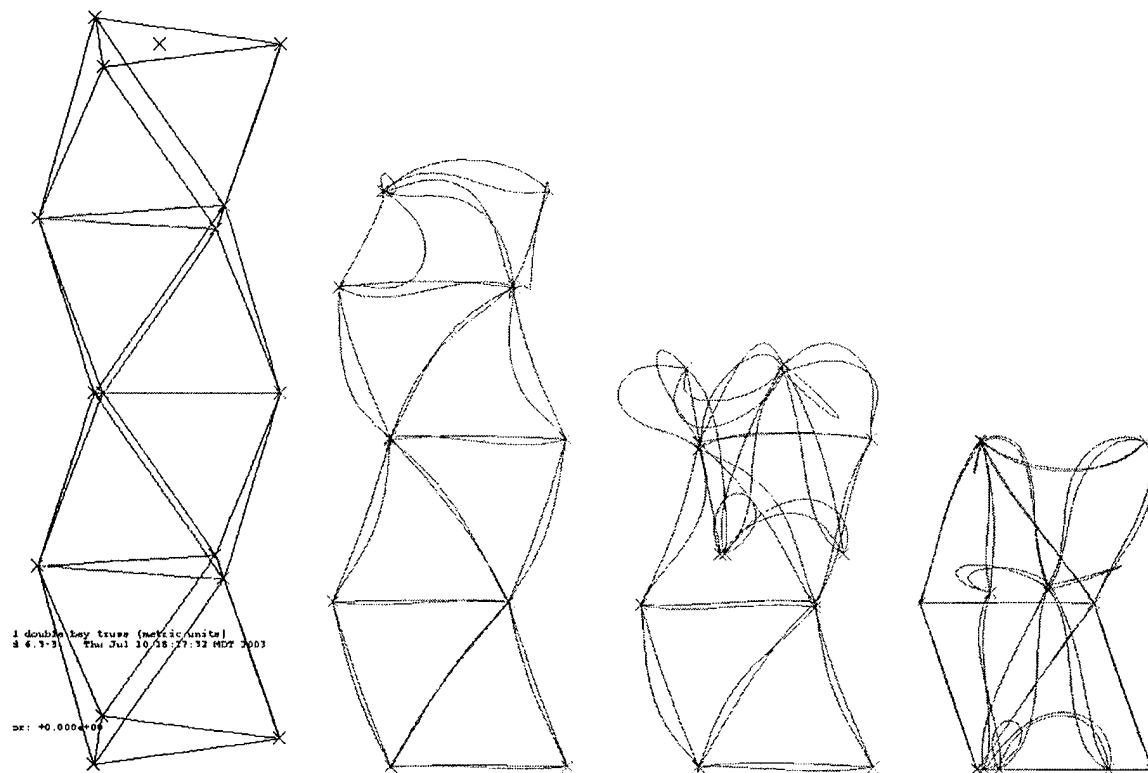
Figure 4-7. Fixed-Free Collapse of Octahedral Truss Architecture
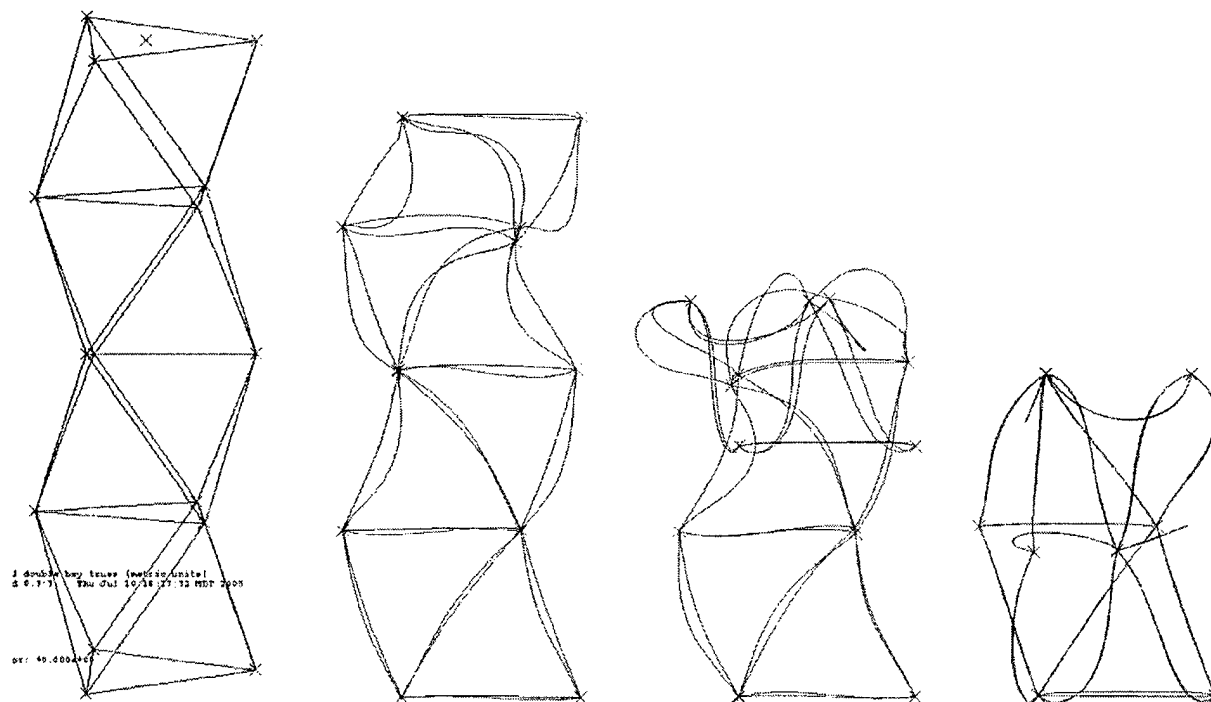
Figure 4-8. Fixed-Sliding Collapse of Octahedral Truss Architecture
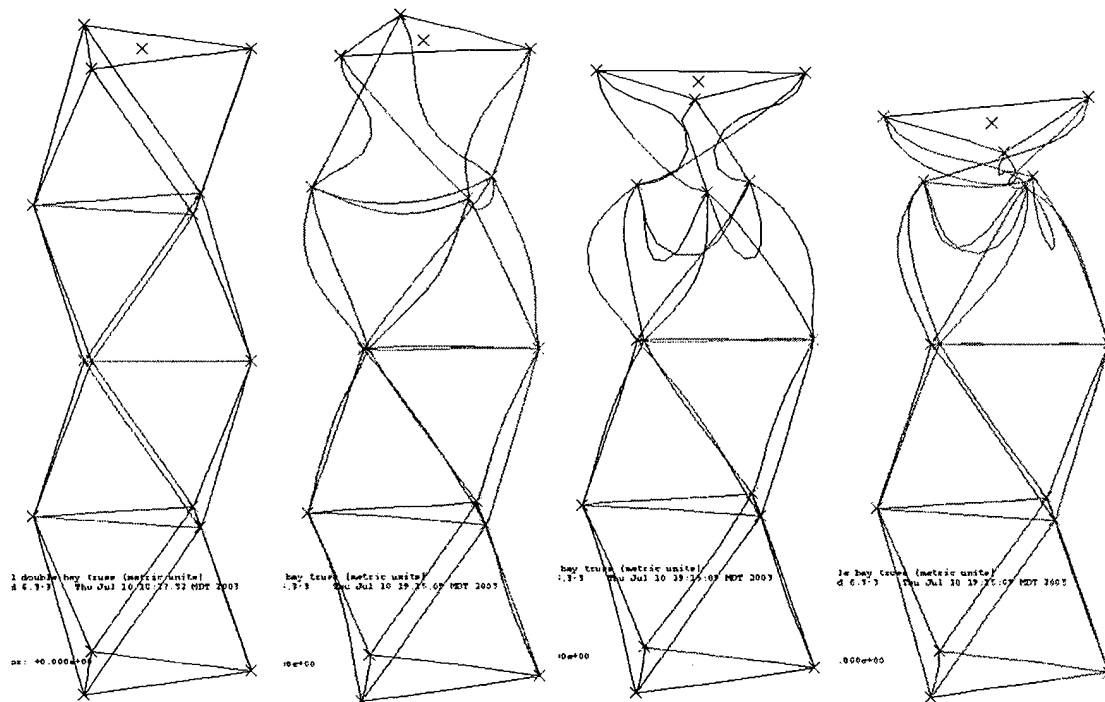
Figure 4-9. Torsional Collapse of Octahedral Truss Architecture

The frame-like connectivity of MEDLS structures leads to strong coupling between adjacent members. This is in contrast to a classical pin-jointed truss in which each member would be able to buckle independently of its neighbors. This coupling leads to the majority of complexity of the coupled global/local buckling of the candidate MEDLS structures that were investigated.

4.1.4   Maximum Normalized Curvature and MEDLS Member Sizing

In extending the results of the above collapse analyses, the maximum element curvature was identified as a particularly scalable measure of the strains required for packaging. In particular, the non-dimensional product of the maximum curvature κ and bay length L can be used to identify the relative merit of various collapse modes independent of local and global scaling. A non-dimensional curvature term was defined according to:

$$\alpha \equiv \kappa L$$

Again, this term is independent of the member cross-section properties and global scale of the truss. Since the strain-curvature relationship is given by:

$$\varepsilon = \frac{D\kappa}{2} \; ;$$

the non-dimensional curvature can be used to calculate the minimum slenderness L/D for a given collapse mode and maximum allowable strain. In this case,

$$\frac{L}{D}_{min} = \frac{\alpha}{2\varepsilon_{max}}$$

In the case of the GLEC analyses, the non-dimensional curvature was found to vary between 3.5 and 6.5. For a maximum allowable strain of 1%, this corresponds to minimum slenderness ratios between 175 and 325.

In packaging 2nd order MEDLS assemblies; it may also be of interest to only partially collapse 1st order elements. In order to relate partial collapse to corresponding strain levels, the maximum curvature history was extracted from the ABAQUS collapse analyses. A typical example of this result is shown in Figure 4.10 for the 4-longeron boom. Bays of unit length lead to a one-to-one relationship between the non-dimensional curvature and the value of maximum curvature shown in this plot.
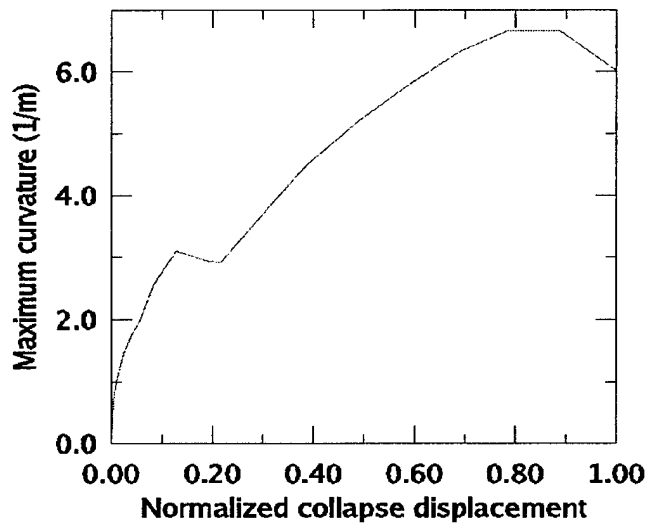
**Figure 4-10. Maximum Curvature During Collapse of the
4-Longeron Two Bay Model**

As shown, the majority of curvature is developed during the first third of the collapse process. As a result, only small reductions in packaging strains may be achieved by reducing the percentage of collapse. It is also interesting to note the local maxima near 10% and 90% collapse. These features were typical of most of the modeled architectures and collapse modes.

4.2   SINGLE ELASTIC LOOP FOLDING

Single Elastic Loop Folding (SELF) is a packaging architecture that works for all lattice structures. The architecture is simple in that it does not rely on a choreographed sequence of simultaneous maneuvers among the truss nodes. To the contrary, each element simply forms a loop; independent of the remaining structure, Figure 4.11. As shown in Figure 4.12, several continuous elements can also be SELF packaged without the need for hinge joints. This independence of motions allows for a great deal of packaging options. The truss can be made to package only in the axial direction as easily as it can be packaged simultaneously or sequentially in other directions. This is illustrated in Figure 4.13, which shows a SELF packaging truss structure supporting a membrane. Further, arbitrary truss configurations can be packaged using SELF because it does not rely on specific deployed truss architecture. In this regard it is very customizable to structures that have not yet been considered.
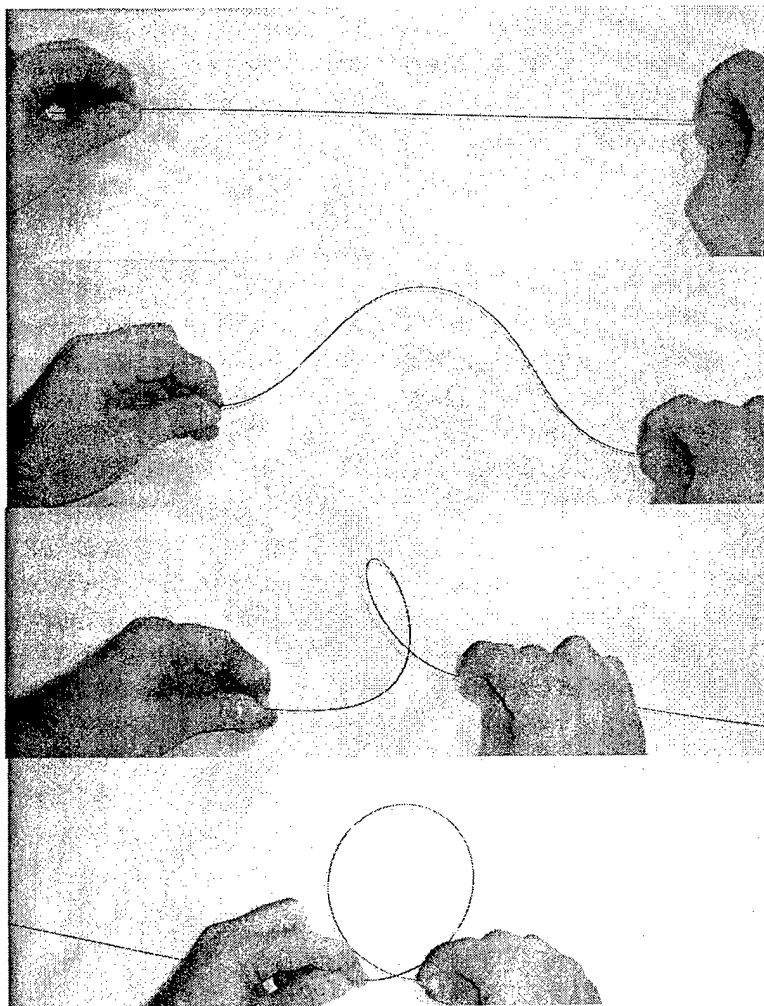
Figure 4-11. Sequence Illustrating SELF Packing of a Single Truss Element
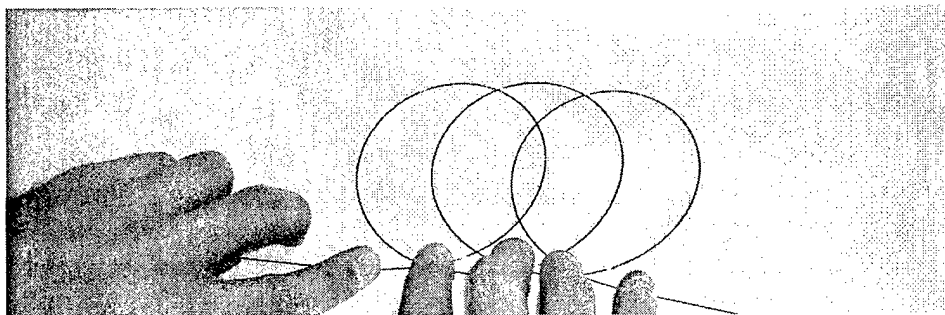
Figure 4-12. SELF Packaging of a Longeron with Several Loops
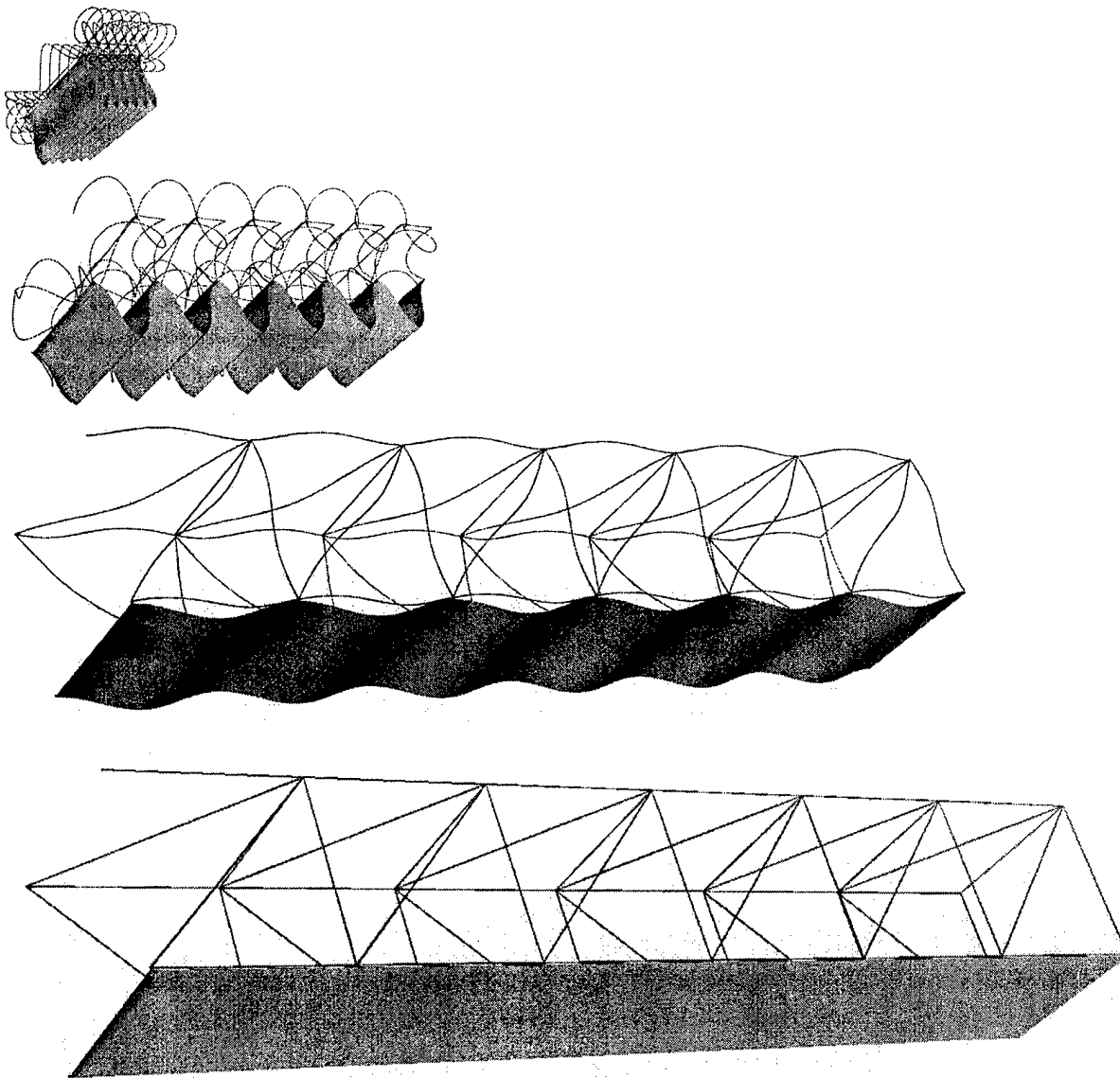
Figure 4-13. Deployment Sequence of a SELF Packaging Truss with Membrane

SELF packaging trusses require an element slenderness ratio of approximately 300 for a material that can strain to 1%. This is a reasonable working strain limit as several high strength graphite/epoxy material systems have failure strains between 1.5 and 2.0% (Hexcel IM9 for example).

SELF packaging is considered to be the lowest strain option for a generic element packaging architecture such that the nodes positions are only scaled in closer to each other. The next best generic element folding

pattern requires elements to be ribbon concertina folded (tear drop profile), however, this results in a significantly higher slenderness of 400.

4.3 HIERARCHICAL TRUSS PACKAGING STUDY CONCLUSIONS

Both the GLEC and SELF approaches appear to offer viable alternatives for packaging MEDLS structures. While the GLEC approach offers the potential for simplified packaging procedures, the analyses performed in the Phase 1 study suggest that additional control of the packaging process may be required. The localization of deformation in GLEC collapse modes would otherwise limit the achievable packaging ratios.

Along with packaging efficiency and simplicity, the necessary strains play a major role in the selection of packaging approaches. Again, both GLEC and SELF offer similar ranges of performance. If member slenderness is to be minimized for a given application, the selection of GLEC architectures and modes may allow slendernesses as low as 175 to be packaged. In addition, both GLEC and SELF approaches could be tailored to focus packaging strains on batten and diagonal elements which could tolerate greater slendernesses without adversely impacting the global performance of the MEDLS element.

5.0 MANUFACTURING CONSIDERATIONS

Even though slender structures are lightest weight structures, engineers traditionally avoid them because they are sensitive to anything that may influence the straightness of the truss elements. In space based MEDLS, element straightness will be influenced by material dimensional stability errors (due to temperature gradients coupled with thermal expansion, residual stresses, moisture gradients and hydroscopic expansion, creep, radiation exposure) and element initial (stress-free) straightness errors. Construction tolerance errors effecting member line-of action may also lead to moments at nodes that result in element curvatures. These are materials and manufacturing issues that, unlike traditional trusses, will require careful consideration and engineering. These issues are discussed here to demonstrate our knowledge of them and our realization that they must be addressed, however, rigorously addressing these issues is left for future work.

The general approach taken in this Phase I Effort was to 1) invent novel elastic packaging architectures and 2) analytically demonstrate the structural viability of these architectures based on a mass efficiency criteria. The rationale being that we should only rigorously consider manufacturing issues if the structures are structurally sound. Due to time and resource constraints during Phase I, we were not able to complete a

detailed assessment of manufacturing issues, however, after completing the two tasks mentioned above, we do believe MEDLS to be an efficient solution to many eminent structural problems and manufacturing issues will be more fully addressed in a Phase II effort. Several of the key manufacturing issues and benefits of MEDLS are discussed below:

MEDLS are envisioned as being constructed from high strength pultruded unidirectional carbon fiber and epoxy resin material systems, e.g. carbon rods. Through independent research and development and flight hardware programs, ABLE is working with pultrusion vendors to develop such rods in a high quality form suitable for slender structures applications.

MEDLS structures share the characteristics of low cost structures; they have relatively few unique parts and each part is required in high volume. Further MEDLS trusses are identically repeated so that they are conducive to automated continuous assembly processes. The challenge of configuring such a process is how to make a strong and reliable (solid) connective joint at the intersection (nodes) of the individual slender members. Ideally, such a process would manufacture such joints in a "one-shot, integral" fashion, avoiding the need to fabricate and join numerous parts with a secondary assembly or bonding process.

A conceivable continuous 1st order MEDLS construction technique is a machine, similar to a screw machine, but with injection molding capabilities. The machine would have continuous spools of carbon rod (one for each diagonal) and hoppers containing the diagonal elements. One bay at a time, the machine could position the elements and place molds on the nodes. The molds would be injected with a structural thermoplastic to form the connection. Chopped fiber reinforcements could be added to the molding material to add strength and improved thermal performance. The molds would then be removed and the process would repeat with the next bay. With such a machine, 1st order MEDLS elements could be made extremely cheap to manufacture in large quantities. An added feature of continuous processing is that quality control also becomes easier to manage, as automated optical inspection techniques could be implemented during the molding process.

6.0   PHASE I STUDY CONCLUSIONS

The following conclusions can be drawn from the completion of the Phase I SBIR study:

1.   Relatively large, lightly loaded (i.e. gossamer) applications are targeted as the best implementation of MEDLS because of the high

member slenderness ratios required to stay within material strain limitations associated with elastic deployment.

2. When the weight increase incurred with a minimum slenderness-restricted truss is greater than approximately two to three, MEDLS are not an efficient architecture, however MEDLS are ideal for gossamer applications where the optimal structure has a slenderness ratio on the order of 300, implying no weight penalty.

3. It was generally found that 2nd order hierarchy is optimal for most reasonable structural requirements. 3rd order hierarchy could be optimal in extremely high stiffness and lightly loaded applications, further study is warranted.

4. The Single Elastic Loop Folding (SELF) method was identified and developed as a feasible and efficient method to package many conceived truss-like architectures. In this unique implementation to MEDLS, a single circular loop is elastically formed in all members, effectively allowing structure stowage without global truss rotations. The SELF method requires high element slenderness ratios, and therefore is limited to very gossamer applications.

5. Global/Local Elastic Collapse (GLEC) involves packaging MEDLS elements through primarily global loading in an effort to simplify the packaging and controlled deployment process. The coupling between global and local elastic buckling provides the opportunity to package 1st order elements by collapsing them through controlled global loading and displacements.

6. Both the GLEC and SELF approaches appear to offer viable alternatives for packaging MEDLS structures. While the GLEC approach offers the potential for simplified packaging procedures, the analyses performed in the Phase 1 study suggest that additional control of the packaging process may be required. The localization of deformation in GLEC collapse modes would otherwise limit the achievable packaging ratios.

7. Necessary strains play a major role in the selection of packaging approaches. Both GLEC and SELF offer similar ranges of performance. If member slenderness is to be minimized for a given application, the selection of GLEC architectures and modes may allow slendernesses as low as 175 to be packaged.

7.0   RECOMMENDATIONS FOR FUTURE WORK

The Phase I effort demonstrated a sound basis for MEDLS within the framework of accepted theoretical mechanics. With that done, it is reasonable to pursue a Phase II SBIR Effort towards developing MEDLS for commercial and defense applications. The objective of a Phase II Effort would be to focus on the hardware development and system implementation of MEDLS technology into a working multiple-bay Engineering Model (EM) of a deployable space structure. The Phase II

Plan will be composed of three efforts. In the first, MEDLS brassboards of several truss configurations will be constructed from super-elastic nickel-titanium alloy. The purpose of these simple models is to develop packaging geometry, kinematics and deployment control methods. Even when restricted to SELF packaging, there are numerous choices for orientation of the loops and the best choice is not immediately obvious. However, with the models it will be very easy to physically investigate all possible orientations and the characteristics of each option will be readily observable. Ideally, several architectures will be constructed including (in order of priority): four longeron truss, three longeron truss, octahedral truss and tetrahedral truss.

Simultaneous to the construction of these models, a pultruded graphite material system and supplier will be selected. ABLE has extensive experience selecting and evaluating the quality of similar unidirectional composite materials and this experience will be utilized. Suitable materials will be acquired and characterized for its suitability to MEDLS applications. The graphite rods will be evaluated for:

o Initial straightness
o Bulk modulus
o Flexural modulus
o Flexural strain limit
o Creep (set taken after storage at high strain and/or elevated temperature for extend period of time)
o Thermal flexure (stiffness/strength behavior at hot/cold extremes)
o Axial load vs. deflection (stiffness and strength)

With the above two tasks complete, a prototype graphite/epoxy MEDLS truss will be designed and built. Design details and issues identified by the Phase I Study will be addressed. Manufacturing considerations identified will be implemented into the development of the assembly process for the model. After manufacture, the prototype will be tested for deployment function as well as static deployed structure stiffness and strength.

In parallel with the hardware development effort, system-level analytical models will be developed that leverage off of the analysis techniques developed in Phase I. These models will be used concurrently during the boom design process to allow system optimization, and will also be used to correlate the prototype EM performance testing.
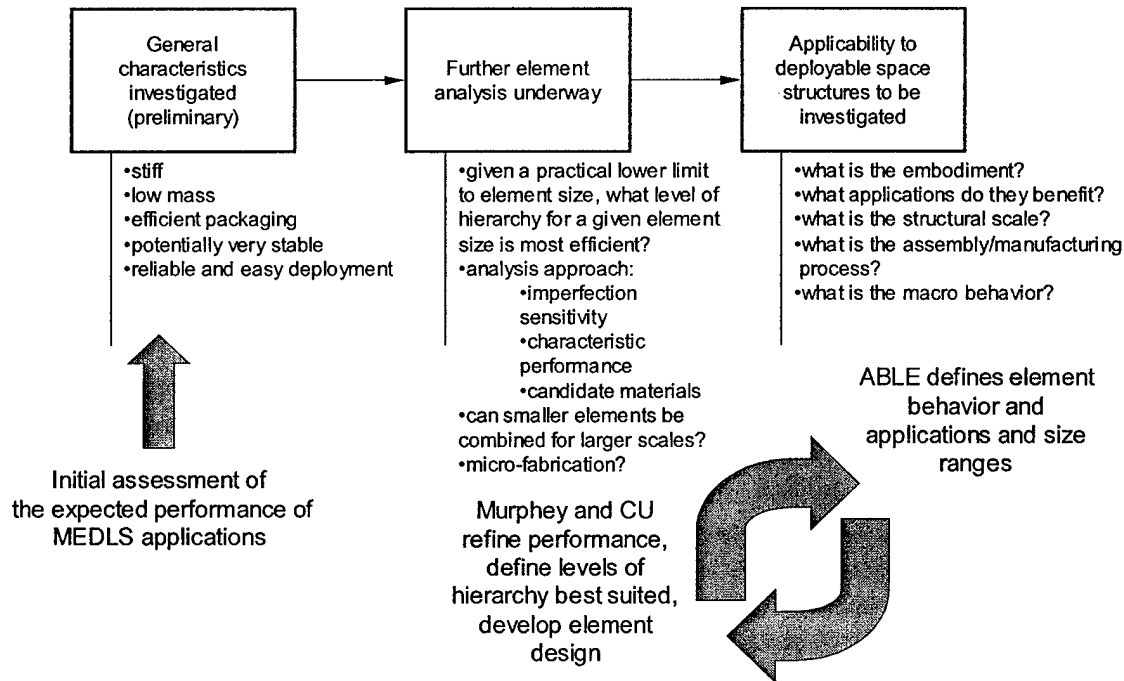
At the end of Phase II, MEDLS will have the physical test data, as well as correlated analytical studies to demonstrate the technology. Such an effort will advance the Technical Readiness Level of MEDLS and ready the technology for implementation into commercial and defense space applications.

**Appendix A
MEDLS Applications Notes**

- <u>M</u>onolithic
  - the idea is to do away with mechanization of any flavor and ideally be homogeneous like a foam at least on the macro scale
- <u>E</u>lastic
  - the structure stores the strain energy for deployment (likely no phase transition or actuators
- <u>D</u>eployable <u>L</u>attice <u>S</u>tructure
  - employing some level beyond typical (if applicable) of structural hierarchy in order to create an efficient deployable truss structure

- Applicability and conclusions so far...
  - more applicable to stiffness driven applications rather than strength

  - higher than 2nd order may have diminishing benefits

  - have a great potential for mass savings and compact stowage

General characteristics investigated (preliminary)	Further element analysis underway	Applicability to deployable space structures to be investigated

•stiff
•low mass
•efficient packaging
•potentially very stable
•reliable and easy deployment
	•given a practical lower limit to element size, what level of hierarchy for a given element size is most efficient?
	•analysis approach:
	  •imperfection sensitivity
	  •characteristic performance
	  •candidate materials
	•can smaller elements be combined for larger scales?
	•micro-fabrication?
		•what is the embodiment?
		•what applications do they benefit?
		•what is the structural scale?
		•what is the assembly/manufacturing process?
		•what is the macro behavior?

Initial assessment of
the expected performance of
MEDLS applications

Murphey and CU
refine performance,
define levels of
hierarchy best suited,
develop element
design

ABLE defines element
behavior and
applications and size
ranges

A list of application ideas (and comments) from 2/28/03 brainstorming meeting at ABLE:

a. **Application:** Perimeter stiffening of a large deployable parabolic reflector (HSC spring-back antenna).

b. **Application:** Backside stiffening (2-D shell-like platform) for parabolic dish or flat SAR reflector with tensioned thin-film (Mylar) reflector element.

c. **Comment:** Use tapered members to optimize deployed MEDLS structural configuration – more uniformly distribute max strain areas of structure. Selective tapering can also be used to enhance packaging.

d. **Comment:** Unidirectional composite rods do not take shear – they yield when twisted, so you need to configure the deployment path of a structure using them to avoid pure torsion of the members.

e. **Application:** "Coilable" mast with longeron and batten components made up of iso-grid type tubes, or MEDLS elements that stow axially and elastically.

f. **Application:** Deployable satellite decoy structure. The MEDLS would deploy and tension a membrane into a space-based structure that has a similar signature as an actual satellite. Multiple (additional) decoys could be packaged within the launch volume of an actual satellite, and deployed into similar orbits.

g. **Application:** Standard or thin-film solar array for small or nano-Sat applications (MEDLS type structures may be more suitable for small deployable panels because these may not be as stiffness driven). MEDLS applied to nano-Sat bus structures.

h. **Comment:** Ideally, the MEDLS basic "building block" element needs to be manufactured as a single part, otherwise the part count gets very high when higher levels of hierarchy are introduced. Look into low-modulus, high strain materials that can be used in a stereo-lithography machine to make the elements. (Can we make our Phase 1 MEDLS element model using stereo-lith?).

i. **Comment:** We should look into using Nitinol (memory metal) in certain MEDLS applications because of its high strain capability.

j. **Application:** Distributed SAR that utilizes a network of small distributed sensors on a gossamer MEDLS platform.

k. **Application:** Apply MEDLS to high altitude balloon structures – less need for complete stowing. Package for shipping to different locales quickly.

l. **Comment:** A primary concern of MEDLS that use their elastic strain energy for deployment is how to control the sequence and rate of deployment. Rate-of-deployment control should be considered as a requirement for certain applications.

m. **Application:** Higher-order hierarchy STEM that utilizes MEDLS and can be rolled up elastically.

n. **Application:** Pantographic deployment for panels (similar to current PUMA edge links) that uses elastic strain energy for deployment force. These elastic deployers can be distributed throughout a structure to increase hierarchy and distribute deployment forces:

o. **Application:** Thermal radiators, sun shields or multi-functional structures requiring low mass and efficient packaging (specific area).

p. **Comment:** Increase packing factor or capability of deploying utilities by better utilizing the interior volume of the coilable configuration with the implementation of MEDLS elements.

q. **Application:** Incorporate the elastic-rigidizable materials into MEDLS applications (hybrid approach) to increase packaging efficiency. Requires heat energy to deploy, but allows better deployment control and enhanced strain performance / packagability.

r. **Comment:** For non-linear packaging analysis to be performed by CU, we should work with them to define the displacement path that nodes of the MEDLS elements take – some predicted stowed shapes that are based on post-buckled shapes would be unrealistic for actual structural configurations. Example: a pure axial deployment of a coilable mast, with no twisting will require unrealistically small longeron elements.

s. **Application:** MEDLS actuator – take advantage of large amount of stored elastic energy in an elastically deformed metal "foam".

Appendix B
Second-Order MEDLS
Optimization Results Examples

Unclassified

3 Longeron, 3 Batten, 3 Diagonal truss MEDLS element optimization results

Material: Graphite/Epoxy, E=29e6 psi; Density=0.06 lb/in^3

Bay Weight Per Length

Unclassified

Bay Radius

Longeron - Longeron Diameter

Longeron - Longeron Length/Diameter
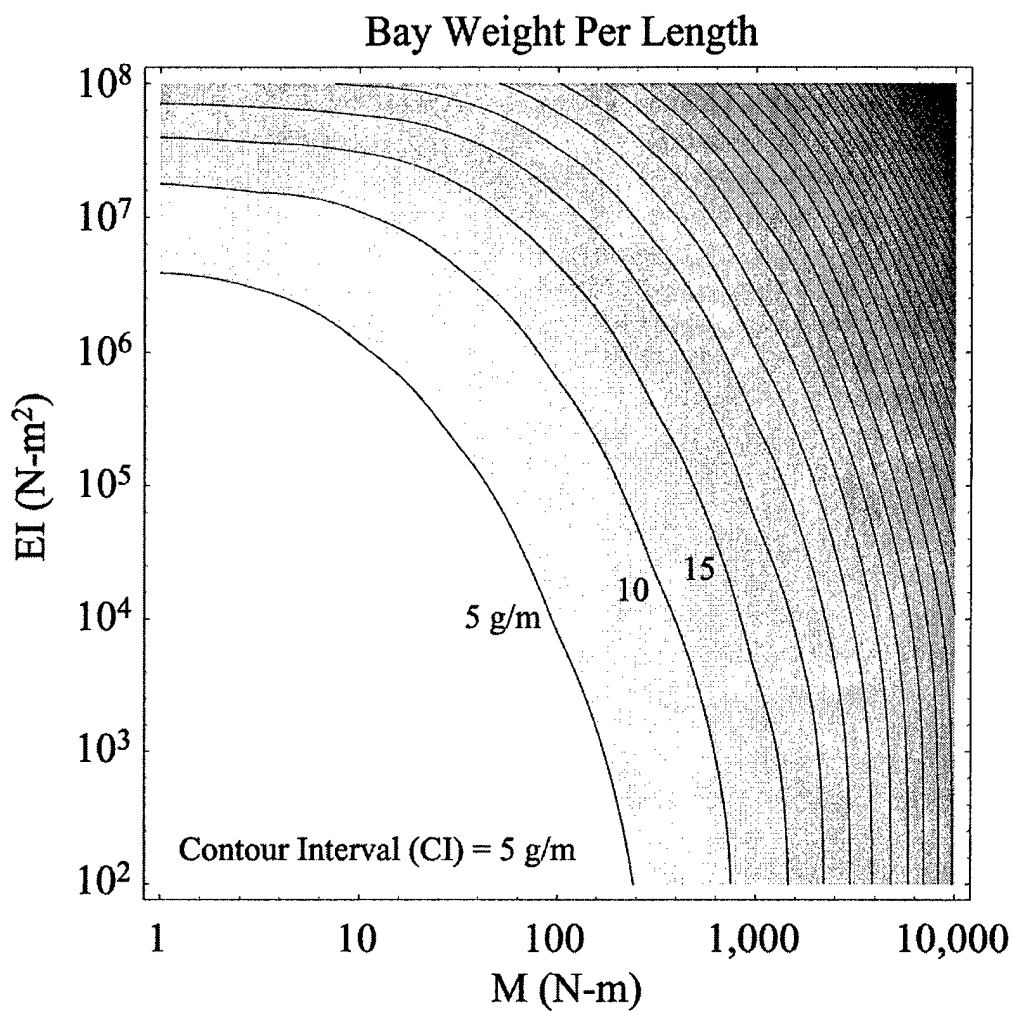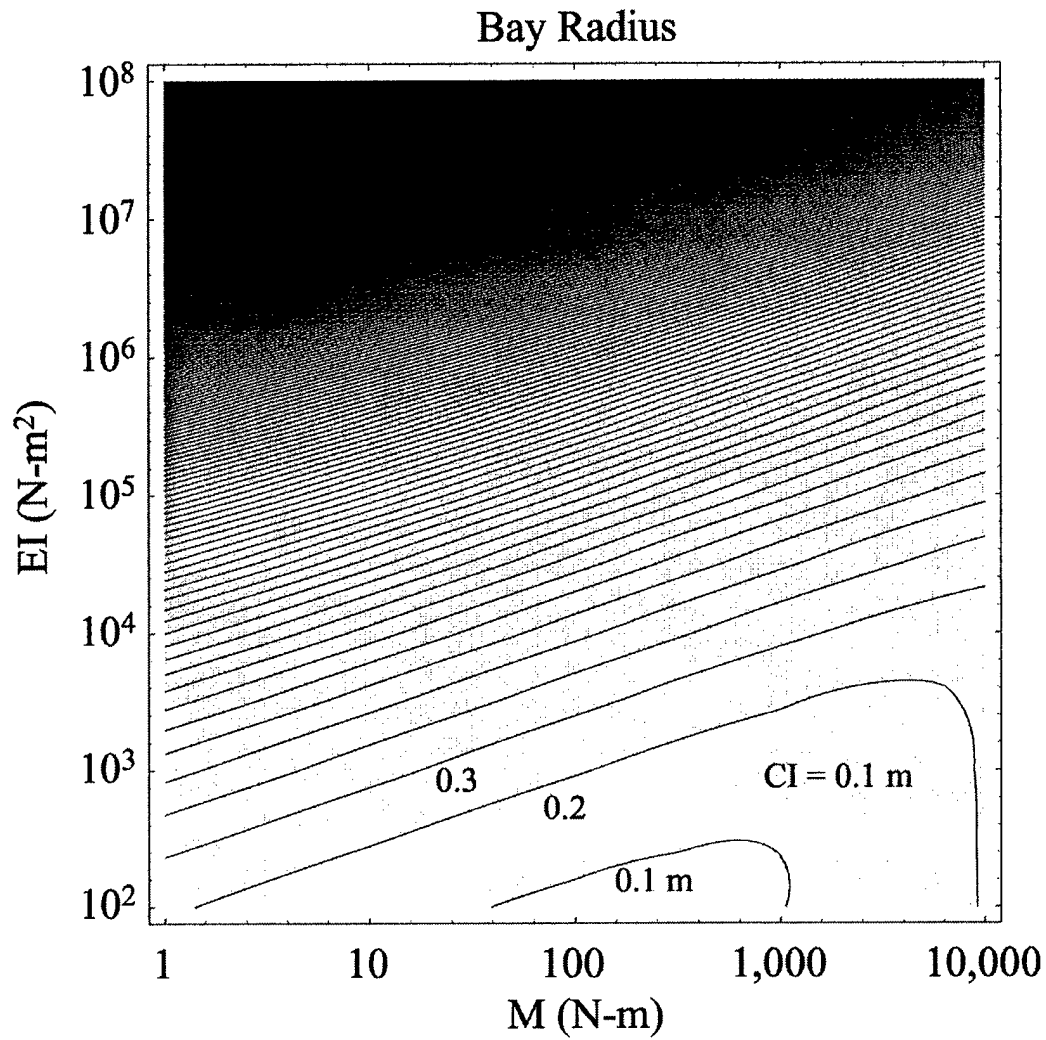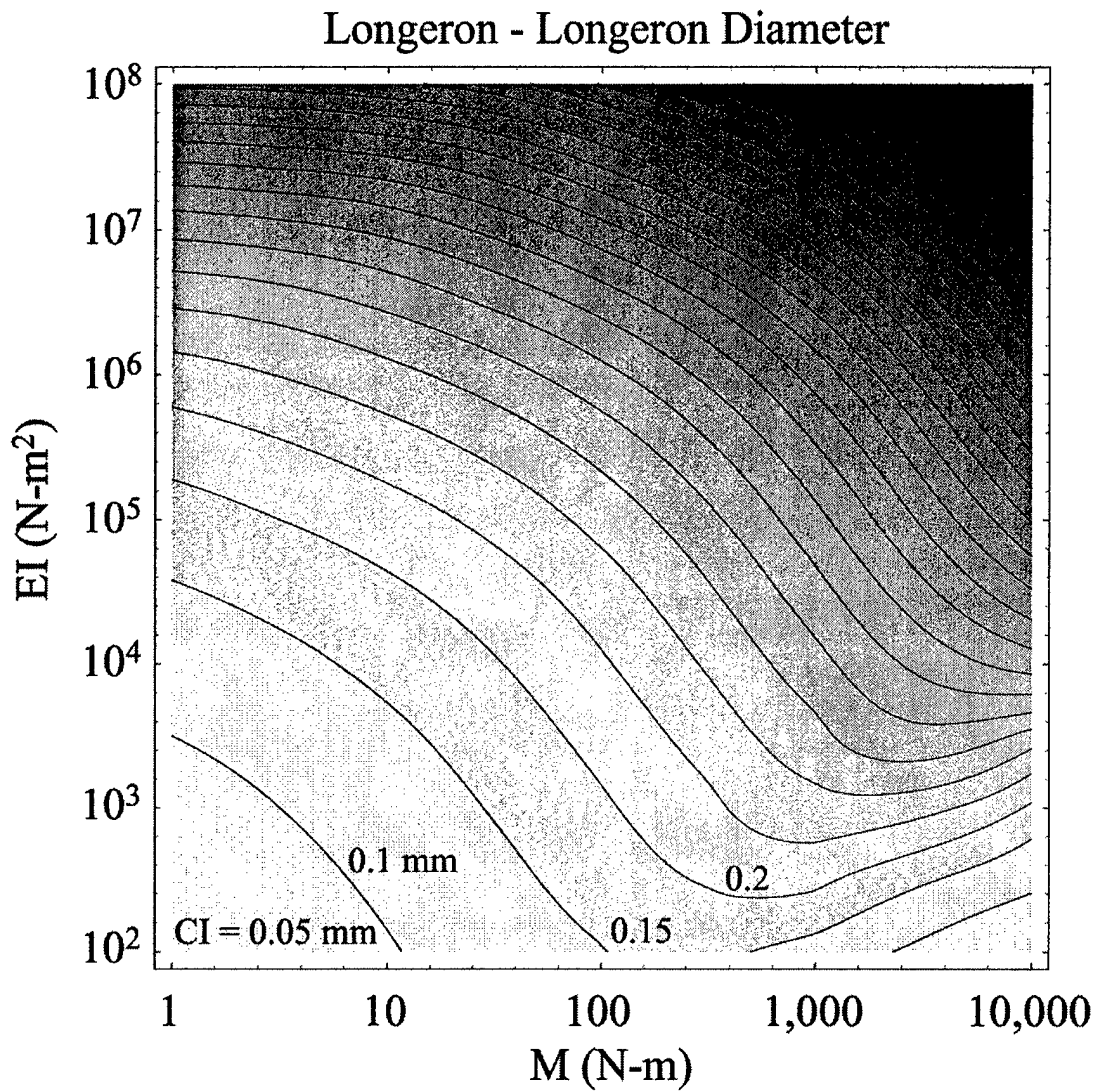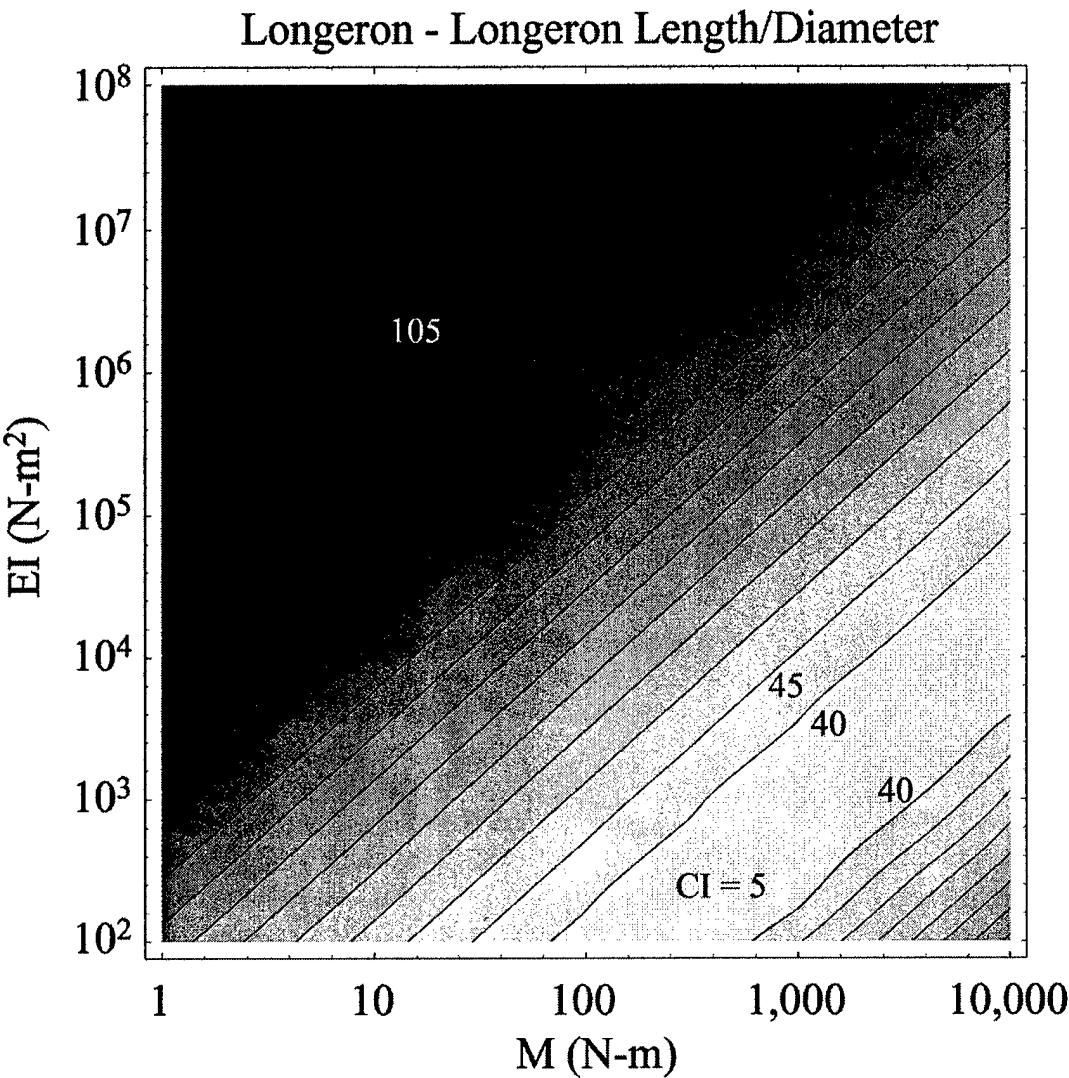
Appendix C
AIAA Paper
"Performance Trends in Hierarchical Truss Structures"
44th AIAA Structures, Dynamics and Materials Conference, 2003

AIAA-2003-1903

SOME PERFORMANCE TRENDS IN HIERARCHICAL TRUSS STRUCTURES

Thomas W. Murphey[*]
Jason D. Hinkle[†]

Abstract

It has been previously demonstrated that increasing structural complexity can lead to lighter weight structures. However, it is not clear that structural complexity or hierarchy enables lighter weight structures for all architectures and load cases. In this paper, the performance trends in linear truss structures are investigated as a function of self-similar hierarchy order and of loading conditions. The investigations show the order of structural hierarchy resulting in a lightest weight self-similar four longeron solid element truss-column is 2nd (a truss made from trusses) for requirements representative of space structures. The resulting truss-column is typically an order of magnitude lighter than the corresponding 1st order truss-column and two to four times larger in diameter. Long and lightly loaded columns are shown to have the greatest potential for mass reduction with increasing hierarchy. Optimization results for 1st and 2nd order self-similar triangular single-laced double-bay trusses subject to bending strength and stiffness requirements are also presented. A comparison of 1st and 2nd order results show a factor of 30 reduction in truss mass and a simultaneous factor of nine increase in truss diameter.

Introduction

There is evidence to suggest that continually increasing levels of structural hierarchy lead to lighter weight, better performing structures. For example, a tube is stronger than a solid rod of equal weight. Likewise, a truss constructed of tubes is stronger than a truss of solid rods of equal weight. Indeed, topology optimization routines often predict highly latticed solutions and can predict hierarchical structures.[1] A benchmark topology optimization problem is a horizontal beam subject to a center load. Solutions to this problem show increased latticing as the mesh refinement is increased (Figure 1) and it is standard

[*] Engineering Consultant, AEC-Able Engineering, Inc., Member AIAA
[†] Research Associate, University of Colorado, Member AIAA

practice to employ filtering techniques to enforce limits on minimum element sizes.

The Eiffel Tower is a popular example of a hierarchical structure, Figure 2. Hierarchy was employed primarily for manufacturing concerns (only relatively short lengths of steel were available at the time), but the resulting structure achieved an unprecedented level of low effective density.[2] In the Eiffel Tower, the lowest order building elements (0th order) are rectangular or L-shaped cross-section bars. Columns are built-up from these elements to form trusses with 1st order hierarchy. These trusses are tied together to build the legs of the tower. Each leg has 2nd order hierarchy. The four legs are tied together to form a tower with 3rd order hierarchy.

In deployable beam-like space structures, 1st order hierarchy is most common and is seen in structures built by AEC-Able Engineering (CoilAble, FastMast, AdamMast and stem tube).[3] Structures with 2nd order hierarchy are also common in the form of trusses built from tubes (AEC-Able SquareRigger solar array, Astro Aerospace AstroMesh antenna, inflatable truss structures by ILC Dover and L'Garde, and the Foster-Miller tubular truss[4]). In these structures, the 1st order hierarchy is a shell structure and the 2nd is a truss. While they have been discussed, no existing space structures with 2nd order hierarchy and latticing at all levels are known.[5] Also, spaces structures with hierarchical order greater than 2nd are unknown to the authors.

From a structural performance perspective, one wonders for what conditions structures of increasing hierarchy offer advantages over lower order structures. Similarly, when advantages are perceived, it is important to know how great they are. This paper investigates such issues in an attempt to provide insight into the performance trends in space structures as their hierarchy is increased. The primary parameter under investigation is n, the order of structural hierarchy.

The first reference to highly hierarchical space structures appears to be in a short essay by the futurologist, Freeman Dyson,[6] however, there are few studies that compare the performance of space structures with hierarchy. Mikulas compared the mass efficiency of several column configurations in his seminal paper on the efficiency of long lightly loaded columns, but he was not specifically looking at

hierarchy.[1] Interest in structural hierarchy is much more prolific from an effective continuum or material perspective. In reference 2, Lakes provides a review of the work in this area and cites 57 records. Lake looked at the efficiency of various space filling trusses in reference 8. Hierarchical materials are common in nature and have inspired the development of new materials.[9,10]

Preliminary Considerations

In this paper, an emphasis is placed on self-similar structures. These are structures which are fractal-like in that they obey the same construction rule with each level of hierarchy. Figure 3 illustrates a two-dimensional truss construction rule. The 0th order element is a solid rod. The 1st order truss is made from 0th order elements and a 2nd order truss is made from 1st order trusses. This process can be repeated *ad infinitum*. Self-similar hierarchical structures are not promoted here as offering a structural advantage over more general hierarchical structures. They are considered here for their analytical simplicity and for their clearly defined hierarchical order.

Consider the longeron element of a 1st order truss. This element behaves as a simply supported column subject to both axial stiffness and axial strength requirements. If this column is a slender solid circular rod, it will fail first in an Euler buckling mode. The element could be made stronger in buckling through redistribution of the existing material in a tubular form, without a reduction in axial stiffness. As the tube radius is increased and the wall thickness is decreased (to maintain constant axial stiffness and mass), the strength continues to increase until local wall buckling becomes the first failure mode. The equations for transition between these regions with increasing load were derived with the assumptions shown in Figure 4,

$$P < \frac{\pi}{4} \frac{(EA)_{req}^2}{EI^e} \qquad \rightarrow \quad Rod$$

$$\frac{\pi}{4} \frac{(EA)_{req}^2}{EI^e} < P < \left(\frac{3\pi}{20}\right)^{\frac{1}{2}} \frac{(EA)_{req}^{\frac{3}{2}}}{IE^{\frac{1}{2}}} \quad \rightarrow \quad Tube \quad (1)$$

$$\left(\frac{3\pi}{20}\right)^{\frac{1}{2}} \frac{(EA)_{req}^{\frac{3}{2}}}{IE^{\frac{1}{2}}} < P \qquad \rightarrow \quad Unclear$$

These equations are graphed in Figure 5 for a high modulus graphite-epoxy material system and length of 1 m. The longeron design requirement parameters (axial stiffness and compressive strength) are given on the horizontal and vertical axes, respectively. The lines on the graph distinguish between regions where a solid rod is adequate (lowest region), a tube is adequate (middle region), and where higher order structural configurations may be more appropriate (highest region). For the rod and tube regions, lines of constant

EA (vertical) are also lines of constant column mass per length.

Transitions to configurations of increasing hierarchical order occur due to a lack of buckling strength in the longeron. For equal diameter and mass, trusses of increasing hierarchy are only needed when lower order structures are strength limited. As a result, the most appropriate structural form at loads above a local wall buckling limited tube is not clear. Reconfiguration of the material as a 1st order truss would cause a reduction in axial stiffness because material used for battens and diagonals do not contribute to the element axial stiffness. This is quite significant as half the mass of a truss structure is typically in battens and diagonals. Reconfiguration of the material into a cellular cross-section (Figure 6) appears to be more efficient since all material contributes to axial stiffness. However, when unidirectional composite materials are used the difference in axial stiffness contribution between truss and cellular architectures may not be very different -- a tube made from unidirectional composite material requires significant portions of material in the transverse direction to resist local wall buckling.

Actually, truss diameter is not typically fixed and the strength and stiffness of a truss can be significantly increased through an increase in both truss diameter (l) and longeron buckling strength (similarly, an increase in diameter allows a reduction in mass if requirements are only maintained). The increase in truss diameter allows the axial stiffness of elements (EA) to be reduced while keeping truss bending stiffness constant ($EI = EAl^2$) and herein lies the ambiguity of hierarchy. For equal mass self-similar trusses, increasing hierarchy tends to continually reduce element axial stiffness. At some point, the truss diameter required to compensate for this reduction in element axial stiffness causes an overall increase in truss mass; an optimal hierarchical order is expected for lightest weight self-similar truss structures.

This process of element axial stiffness reduction is dramatic in trusses. Consider a 1st order self-similar truss construction rule that allocates a fraction, β, of the total truss mass to longerons. For the 2nd order structure, the fraction of mass allocated to axial members is $\beta_{\alpha\beta} = \beta^2$. In general,

$$\beta_{\alpha i} = \beta^i \qquad (2)$$

is allocated to axial material. A 3rd order self-similar truss with $\beta = 0.5$ allocates only 12.5% of material to axial stiffness.

The above considerations show that performance trends with increasing hierarchy are not always obvious; regardless of cost and manufacturing

considerations, an increase in hierarchy is not always better.

A Simple Example

To illustrate analytically the trends in hierarchical structures, a self-similar hierarchical four longeron truss-column is investigated. This truss is chosen because it is representative of real structures and can be analytically modeled with simple equations. The following characteristics are assumed:

- The truss has four longerons, four member batten frames and four diagonals per bay.
- Each bay is identical (a regular truss).
- Elements are pin jointed.
- Bays are cubic with edge dimension, l.
- All lowest order elements have identical solid circular cross sections characterized by a diameter, d.

Four bays of a 2nd order example are shown in Figure 7. All 1st order bays in the 2nd order structure are assumed identical and a similar approach is taken for higher orders. For this truss, a parameter that is useful for characterizing the weight per length is the total length of elements per length of bay,

$$\mu = \frac{4l + 4\sqrt{2}l + 4l}{l} = 13.66 \qquad (3)$$

For this truss, the axial material fraction is found as $\beta = 4/\mu = 0.29$.

The dimensions of lightest weight hierarchical trusses subjected to axial compressive loads are now derived. The standard procedure for high strength slender structures, and that which is used here, is to equate global and local buckling modes. In doing so, a column length (L) is introduced for the global buckling mode. With the above assumptions, only two design variables are required to specify a 1st order truss (l and d). This is convenient since there are two design equations (global and local buckling) for sizing the column.

In the following, this sizing procedure is carried out for the hierarchical truss from 0th order (single solid rod) to higher orders and, upon identifying patterns in the resulting performance equations, equations as a function of n are written. A notation in the form of x_{ij} or x_i is used. i indicates the top level hierarchical order of the truss and j references an element of jth order of the truss ($j \le i$).

Consider the 0th order solid rod with area (A_{00}) and cross-section moment-of-inertia (I_{00}) given by,

$$A_{00} = \frac{\pi}{4}d_0^2, \quad I_{00} = \frac{\pi}{64}d_0^4 \qquad (4)$$

where d_i is the element diameter. Weight per length of the rod ($w_{00} = \rho A_{00}$) is given by,

$$w_{00} = \rho\frac{\pi}{4}d_0^2 \qquad (5)$$

where E is Young's modulus and ρ is the bulk material weight density. The Euler column buckling strength (P_{00}) of this element is,

$$P_{00} = \frac{\pi^2 EI_{00}}{L^2}. \qquad (6)$$

The lightest weight structure is found by solving Equation (6) for the single design variable, d_0, with $P_{00} = P$,

$$d_0 = \left[\frac{64}{\pi^3}\frac{PL^2}{E}\right]^{\frac{1}{4}} \qquad (7)$$

The optimized rod weight per length is found through substitution of this design variable solution into Equation (5),

$$w_{00} = \rho\left(\frac{4}{\pi}\frac{PL^2}{E}\right)^{\frac{1}{2}} \qquad (8)$$

A 1st order truss composed of 0th order solid rods is now similarly sized. The truss will have 0th and 1st order element cross-section moment-of-inertias given by,

$$I_{10} = \frac{\pi}{64}d_1^4, \quad I_{11} = A_{10}l_0^2 \qquad (9)$$

where A_{10} is the area of a single longeron,

$$A_{10} = \frac{\pi}{4}d_1^2 \qquad (10)$$

The truss-column weight per length is given by,

$$w_{11} = \rho\mu A_{10} \qquad (11)$$

and will have global buckling strength (P_{11}) and local buckling strength (P_{10}) given by,

$$P_{11} = \frac{\pi^2 EI_{11}}{L^2}, \quad P_{10} = \frac{\pi^2 EI_{10}}{l_0^2} \qquad (12)$$

The lightest weight truss is found by solving Equations (12) for the two design variables, l_0 and d_1, with $4P_{10} = P_{11} = P$,

$$l_0 = \left(\frac{1}{\pi^2}\frac{PL^2}{E}\right)^{\frac{1}{4}}, \quad d_1 = \left(\frac{8}{\pi^3}\frac{PL}{E}\right)^{\frac{1}{3}} \qquad (13)$$

Substitution of these design variables into Equation (11) gives,

$$w_{11} = \rho\mu\left(\frac{1}{\pi^4}\frac{P^2L^2}{E^2}\right)^{\frac{1}{3}} \qquad (14)$$

A similar sizing process was carried out for higher orders of hierarchy. With each order, an additional

3
American Institute of Aeronautics and Astronautics

buckling equation and design variable are added so that the system of equations is sufficient to size the structure. This process was carried out sufficiently (6th order) to identify the patterns with increasing n. The resulting general equations for element diameter, longest longeron length and structure weight per length are:

$$d_n = \left[2^{3+n-n^2} L\left(\frac{P}{\pi^3 E}\right)^{\frac{n+1}{2}}\right]^{\frac{1}{n+2}} \quad (15)$$

$$l_{n,n-1} = \left[2^{1-n} L^{n+1}\left(\frac{P}{\pi^3 E}\right)^{\frac{1}{2}}\right]^{\frac{1}{n+2}} \quad (16)$$

$$w_{n,n} = \rho L^2\left[\frac{2^{3(1-n^2)}}{\pi^{2n+1}} L^2\left(\frac{P}{E}\right)^{n+1}\right]^{\frac{1}{n+2}} \quad (17)$$

The column weight per length, which is graphed in Figure 8, has a minimum with respect to n that typically ranges from two to four. The highest order element length (which is approximately equal to column diameter) and the smallest element diameter equations are graphed in Figures 9a and 9b, respectively.

It is of practical interest to determine the relative truss weight reduction with increasing n and the parameters the optimal n is most sensitive to. The ratio of the weight per length of a truss of order $n+1$ to a truss of order n reveals this.

$$\frac{w_{n+1}}{w_n} = \mu\left[2^{-3+\frac{6}{6+5n+n^2}}\right]\left(\frac{P}{\pi^3 EL^2}\right)^{\frac{1}{6+5n+n^2}} \quad (18)$$

When this ratio first exceeds unity, increasing hierarchy just begins to increase the weight of the structure. This point is the optimal n. Equation (18) is a function of n and the single non-dimensional material, geometry and load parameter $P/(EL^2)$ and is graphed in Figure 10 for $\mu = 13.66$. The graph shows hierarchy is increasingly beneficial as the non-dimensional parameter decreases. It is only for very long and lightly loaded applications that a hierarchy greater than two results in lighter weight columns. For example, the load supported by a 100 m (328 ft) column with $E = 200$ GPa (29 Msi) should be less than 2 N (0.45 lb) before 3rd order hierarchy is employed. The load below which the same column is optimally 2nd order is 2.0E6 N. Considering a short column of 1 m, the load should be less than 200 N for 2nd order hierarchy to be optimal. These results seem to indicate 2nd order hierarchy is optimal for most reasonable columns.

Where appropriate, an increase in hierarchy from 1st order to 2nd order typically corresponds to a factor of 10 mass reduction and a factor of two to four increase

in diameter. The weight reduction and diameter ratio increase as the non-dimensional parameter decreases.

General Optimization

While they characterize general trends, Equations (15) through (18) may not be representative of efficient space structures. First, space structures are more generally subject to bending stiffness and strength requirements as opposed to column requirements (columns are often considered with space structures because the elements of trusses are columns). Second, structures are not typically limited to the constraints of cubic bays and uniformly equal element diameters. Despite these limitations, the previous example analytically demonstrates general trends with increasing hierarchy and shows the existence of minima in these trends. In the following, more robust numerical optimizations of self-similar trusses are considered for 1st and 2nd order hierarchy. Restrictions on the similarities between elements within a bay are removed and strength and stiffness requirements are more accurately calculated. Straightness imperfection effects are also included. These optimizations are carried out for requirements spanning several orders of magnitude so that the trends are revealed for a large range of space structures.

Triangular Truss Construction Rule

A three longeron regular truss construction rule is assumed. This truss has been called a triangular double-bay single-laced truss and is shown in Figure 11. The geometry of the truss is characterized by two parameters, bay length (l) and radius (r). The batten and diagonal lengths are related to these parameters by,

$$b = r\sqrt{3} \quad (19)$$

$$d = \sqrt{l^2 + b^2} = \sqrt{l^2 + 3r^2}$$

The longeron, diagonal and batten stiffness properties are characterized by their effective axial spring stiffnesses: k_l, k_d and k_b, respectively. The diagonals of successive bays are assumed to be laced in alternating directions so that the simplest repeating element has length $2l$. The triangular truss was chosen because it is simple and commonly used in engineering structures. Solid circular 0th order elements are assumed. The details of nodes (joints where longerons, diagonals and battens intersect) are not considered.

Requirements

The general set of 12 global requirements of a typical beam (stiffness and strength for: axial loads, torsional loads and bending and shear loads about two axes) are reduced to two. First, torsion and axial loads are not considered (-4) because the structure is assumed to act primarily in a bending sense. The beam is also assumed to be isotropic so that bending and shearing are respectively equal for both axes (-4). This leaves

four parameters (bending and shear stiffness and strength). For a cantilever beam, bending and shear are not independent and this is used to map shear requirements into bending requirements (-2). This reduces the optimized beam requirements to two: bending stiffness and bending strength. For trusses, all lower order elements act as columns and have requirements derived from the global structure requirements.

Bending Stiffness

A cantilever beam is assumed for mapping shear requirements onto bending requirements. The tip displacement (δ) of a cantilever beam is due to both bending and shearing compliances,

$$\delta_{EI} = \frac{PL^3}{3EI}, \quad \delta_{GA} = \frac{PL}{GA}$$

$$\delta_{total} = \delta_{EI} + \delta_{GA} = PL\left(\frac{L^2}{3EI} + \frac{1}{GA}\right) \tag{20}$$

An effective bending stiffness (EI_{ef}) can be defined that gives δ_{total} when used in the equation for δ_{EI},

$$\delta_{EI_{ef}} = \delta_{total}$$

$$\frac{PL^3}{3EI_{ef}} = PL\left(\frac{L^2}{3EI} + \frac{1}{GA}\right) \tag{21}$$

$$EI_{ef} = \left(\frac{1}{EI} + \frac{3}{GAL^2}\right)^{-1}$$

In the current work, L is assumed to be equal to 10 times the column diameter. This procedure ensures that an optimized beam composed of many bays does not avoid the bending stiffness requirement by deforming through a shear compliance.

Effective Continuum Stiffness Equations

The analysis of hierarchical structures is potentially daunting because the number of elements increases exponentially with hierarchical order. In the present work, the problem is kept manageable through the assumption of regular trusses. In regular trusses, a relatively simple bay that identically repeats itself can be identified. The behavior of a beam constructed from many bays is then only a function of the behavior of a single bay. Thus, simplified effective continuum equations representing the behavior of a beam of arbitrary length can be written as functions of single bay geometry and element properties.

The direct stiffness method of matrix structural analysis was implemented symbolically in a Mathematica (Wolfram Research Corporation) program to derive such equations. The program generates symbolic equations for the deflections and loads in arbitrary three dimensional pin jointed truss structures. This has been applied to the triangular truss and shown to yield results identical to those obtained by Renton

and Noor.[11,12] The program is readily adaptable to future structural configurations that may become of interest. The effective continuum stiffness equations for the triangular truss are,

$$EI = \frac{3}{2}k_i h r^2$$

$$EA = 3k_i l$$

$$GJ = \frac{9k_i k_i r^4}{4l^2(k_d + k_i) + 12k_i r^2} \tag{22}$$

$$GA = \frac{18k_i k_i r^2}{l^2(k_d + 4k_i) + 12k_i r^2}$$

Strength Equations

The strength equations are dependent on whether the structure under consideration is the highest order bay (subject to a bending strength requirement) or a lower order element (subject to column strength requirements). Regardless, the structures are subject to both multi-bay and single bay requirements.

Multi-bay strength requirements. A column element must have enough combined bending and shear stiffness to resist global buckling. The equation for global buckling with combined bending and shear compliance is,[13]

$$P_{cr} = \frac{P_c}{1 + \frac{P_c}{GA}} \tag{23}$$

where,

$$P_c = \frac{\pi^2 EI}{L^2} \tag{24}$$

The battens and diagonals must provide enough support to the longerons to enforce local buckling. Also in reference 13, Timoshenko derived an expression for the minimum spring stiffness of equally spaced supports on a long column required to ensure the first buckling mode has a wavelength equal to the support spacing,

$$\alpha = 4\frac{P_e}{l} \tag{25}$$

where P_e is the Euler buckling load of the column with length equal to the distance between supports. For the current truss, α is assumed equal to the shear stiffness per length of the face of a bay,

$$\alpha = \frac{b^2 k_d k_i k_i}{l^2(k_b + k_d)k_i + k_i(k_d + k_i)l^2} \tag{26}$$

where b is the length of a batten.

Single bay strength requirements. For pure column loads and bending moments, only the longerons are stressed; diagonals and battens are not loaded and hence, not subjected to strength requirements. The longerons of a column must be stiff enough to resist local buckling and this strength is predicted with

Equation (23). The load a longeron must support is given by,

$$F_i = \frac{P}{3} \quad (27)$$

where P is the compressive load applied to the column. For bending loads longerons are stressed according to,

$$F_i = \frac{M}{2r} \quad (28)$$

As mentioned, shear strength requirements are mapped into a bending strength requirement. A cantilever beam of length L and root moment M has tip load (maximum shear, V) given by,

$$V = \frac{M}{L} \quad (29)$$

A shear load causes the following maximum element loads,

$$F_i = \frac{2VL}{r3\sqrt{3}}$$
$$F_d = \frac{2V\sqrt{L^2 + 3r^2}}{r3\sqrt{3}} \quad (30)$$
$$F_i = \frac{2V}{3\sqrt{3}}$$

Equation (23) is used to ensure each element is stiff enough to support the load given by Equation (30).

A material strength limit of 138 MPa (20 Ksi) is also enforced for all elements. This constraint becomes active only for very low bending stiffness and high bending strength requirements.

Geometric Imperfection Effects

As element slenderness (length to diameter ratio) increases, straightness imperfections can degrade axial stiffness. An analysis of this stiffness degradation has been previously derived for an initially curved beam-column and the solution is applied here.[18,19] Imperfections are approximated as a sinusoid with half wave-length l and amplitude a. It can be shown that the effective spring stiffness of this element is,

$$k_o = \frac{E_c A}{l} = \frac{P_{cr}}{2\varepsilon_o l} \quad (31)$$

where P_{cr} is the Euler buckling load of the element and ε_o is the effective shortening strain due to the imperfection,

$$P_{cr} = \pi^2 \frac{EI}{l^2}$$
$$\varepsilon_o = \frac{1}{1 + \frac{4l^2}{\pi^2 a^2}} \approx \frac{\pi^2}{4}\left(\frac{a}{l}\right)^2 \quad (32)$$

The imperfection amplitude to length ratio (a/l) can be attributed to a dimensional stability strain error (potentially due to thermal or manufacturing effects).

A uniform outer fiber strain error (ε) causes a slender rod to take the shape of a circle. This circle can be approximated as a parabola so that the rod imperfection amplitude to length ratio is given by,

$$\frac{a}{l} \approx \frac{\varepsilon}{8}\frac{l}{r} = c\frac{l}{r} \quad (33)$$

In the current work, the strain error is assumed to be 0.012% so that $c = 0.0000015$. This corresponds to a 120 °C temperature change in a unidirectional graphite material with coefficient of thermal expansion of $1\,\mu\varepsilon/°C$. This imperfection is assumed for all elements and all hierarchical levels.

The total effective spring stiffness of an element is that due to a combination of stretching and bending,

$$k_o = \left[\frac{1}{k} + \frac{1}{k_b}\right]^{-1} \quad (34)$$

For a solid rod element, k is given by,

$$k = \frac{EA}{l} \quad (35)$$

A reduction in local buckling strength occurs in imperfect columns due to non-uniform longeron loading. This strength reduction mechanism is not considered here.

Design Variables

The optimization method considers a beam of arbitrary length subject to both bending stiffness and bending strength requirements and finds the lightest weight per length bay that satisfies the requirements. For the 1st order system, bay length and diameter are optimized along with longeron, batten and diagonal diameters (five parameters total). The effective beams are assumed regular in that member diameters do not vary along their length. For the 2nd order system, bay length and diameter are optimized for the 1st and 2nd order elements (eight parameters) along with (independently) the diameters of the three 1st order elements of the three 2nd order elements (nine parameters) for a total of 17 optimization parameters. This optimization procedure was carried out for a range of bending stiffness and strength requirements.

Results

The 1st and 2nd order triangular truss optimization results for weight per length, bay radius, 0th order longeron diameter and 0th order longeron slenderness are shown in Figures 12 and 13, respectively.

1st Order Optimization Results

Below an EI to M ratio of 100 m, truss weight per length and longeron diameter are independent of EI. In this region strength requirements drive the truss design. Above an EI to M ratio of 10^5 m the

truss is more stiffness driven and the strength requirements are not active.

Longeron slenderness exhibits two regions of relatively constant values of 30 and 115. The transition between the two regions is relatively narrow but corresponds to the cases where both stiffness and strength constraints are active. This interesting result implies structures should be either slender ($l/d = 115$) or stout ($l/d = 30$), determined by the ratio of EI to M.

2nd Order Optimization Results

The 2nd order truss optimization results exhibit similar regions of independence of EI and M requirements, but the transition region is much broader. Both EI and M constraints are active over most of the plots. Interesting minima are observed on the longeron diameter contours lines.

1st to 2nd Order Truss Trends

The ratio of 2nd order truss weight per length to 1st order truss weight per length is shown in Figure 14. The optimized 2nd order structure weight per length ranges from 0.010 to 0.035 times that of the equivalent optimized first order structure. The greatest weight savings is for 1st order structures that are farthest from a point where both stiffness and strength constraints are active. Thus, structures with high EI and low M requirements or low EI and high M requirements receive the greatest weight reduction with an increase to 2nd order hierarchy.

The ratio of 2nd order truss diameter to 1st order truss diameter is graphed in Figure 15. While this ratio ranges from 1 to 14, there is a broad plateau at 9. Thus, an optimized 2nd order truss is typically an order of magnitude larger than an optimized 1st order truss.

Conclusions

Space structures often have stringent size requirements and are lightly loaded. Such structures are more stiffness limited than strength limited, precluding the need to incur the complexities of increased hierarchy.[20] However, when an increase in diameter is allowed, hierarchy greater than 1st improves performance. In the assumed solid element truss-column, the optimal hierarchy is typically 2nd. A long, lightly loaded truss-column has the greatest potential to benefit from an increase in hierarchy. The weight efficiency of the hierarchical truss was shown to be a function of the non-dimensional parameter $P/(EI^2)$. The transition from 1st to 2nd order hierarchy occurs for for values of this parameter near 10^{-9}. The transition from 2nd to 3rd order hierarchy occurs for values of this parameter near 10^{-15}. Very large solar sails are potentially in the load range where 3rd order hierarchy results in the lightest weight structure.

General optimization of 1st and 2nd order solid element trusses subject to bending requirements indicate optimized 2nd order trusses are two orders of magnitude lighter than 1st order trusses. They are also typically nine times larger in diameter. These results differ significantly from those for the column optimizations. It is hypothesized that the differences are due to the change in requirements (bending vs. compressive strength) and a higher effective axial mass fraction (β) for the more generally optimized beams.

Several assumptions were made in these studies that have the potential to influence the results. For example, solid element as opposed to tubular element structures were assumed. The structures were assumed self-similar trusses as opposed to allowing lattice and shell architectures in the same structure. Relatively non-conservative imperfection amplitudes were assumed. Also, a high modulus, low density and high strength material system was assumed. These are all interesting aspects of the current study and warrant investigation. It is hoped that such can be accomplished in future work.

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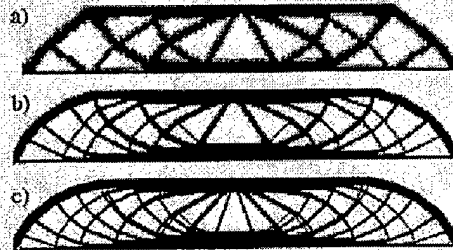
Figure 1: Increased latticing in topology optimization solutions for a beam with center load as mesh density is increased (reference 1).
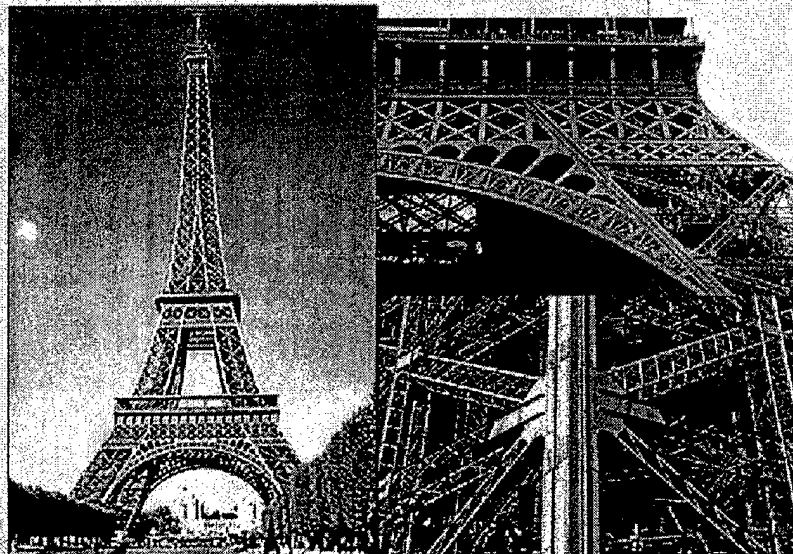
Figure 2: Photographs showing the 3 levels of hierarchy in the Eiffel Tower (Paris, France).
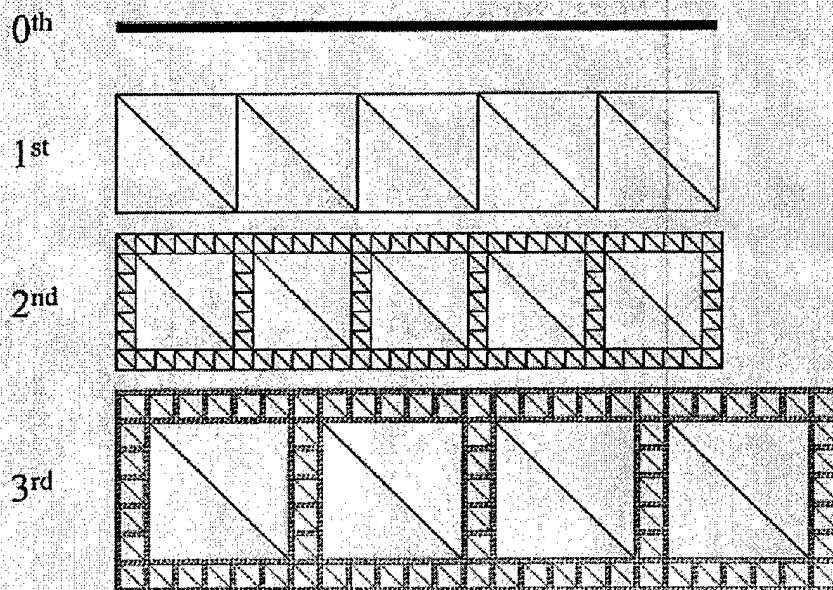
0th

1st

2nd

3rd

Figure 3: Increasing structural hierarchy with a two-dimensional self-similar construction rule.

Rod to tube transition equation derivation:

Cross-section area and inertia: $A = \frac{\pi}{4}d^2$ and $I = \frac{\pi}{64}d^4$

Axial stiffness requirement determines required d: $EA = (EA)_{req} \rightarrow d^2 = \frac{4}{\pi}\frac{(EA)_{req}}{E}$

Transition load determined by Euler buckling constraint: $P_{trans} = \frac{\pi^2 EI}{l^2} = \frac{\pi}{4}\frac{(EA)_{req}^2}{El^2}$

Tube to higher order transition equation derivation:

Cross-section area and inertia: $A = \pi dt$ and $I = \frac{\pi}{8}d^3 t$

Axial stiffness and global buckling requirements determine tube thickness and diameter:

$$\left[\begin{array}{l} P_{trans} = \frac{\pi^2 EI}{l^2} = \frac{\pi^3 Ed^3 t}{8l^2} \\ (EA)_{req} = E\pi dt \end{array} \right] \rightarrow \left[\begin{array}{l} d = \frac{2l}{\pi}\sqrt{\frac{2P_{trans}}{(EA)_{req}}} \\ t = \frac{1}{2El}\sqrt{\frac{(EA)_{req}^3}{2P_{trans}}} \end{array} \right]$$

Transition load determined by local buckling constraint:

$$P_{trans} = A\frac{12}{10}\frac{Et}{d} \rightarrow P_{trans} = \left(\frac{3\pi}{20}\right)^{1/2}\frac{(EA)_{req}^{3/2}}{lE^{1/2}}$$

Figure 4: Rod-tube-higher order column transition equation derivations.
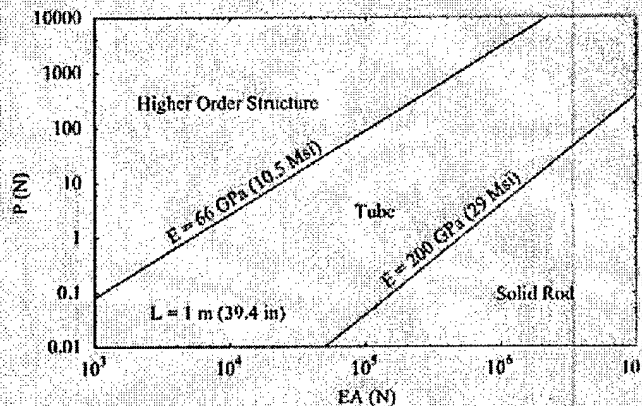
Figure 5: Simplest longeron form as a function of stiffness and strength requirements.
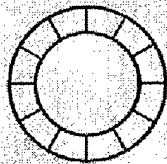
Figure 6: A 2nd order cellular beam cross-section.
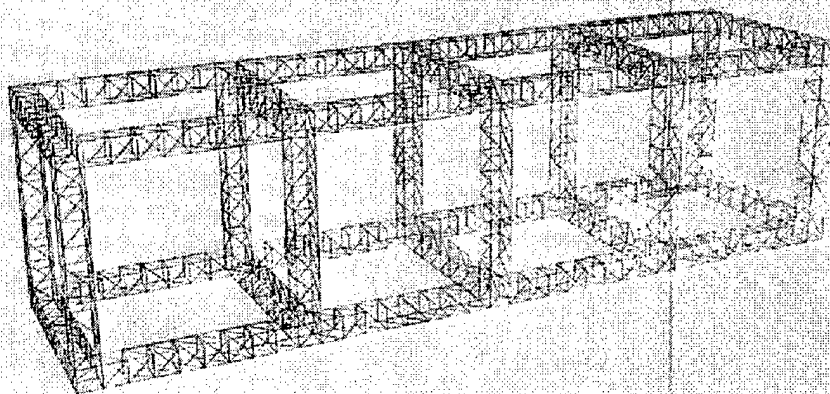
Figure 7: 2nd order truss-column (2nd order diagonals not shown).
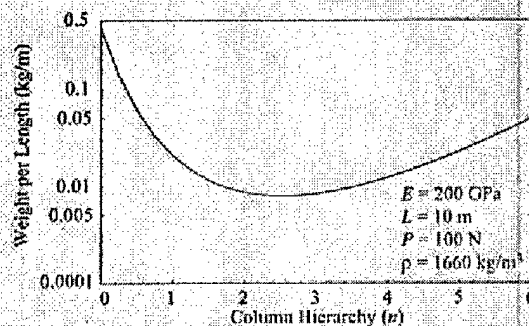

Figure 8: Column weight per length as a function of hierarchy order.
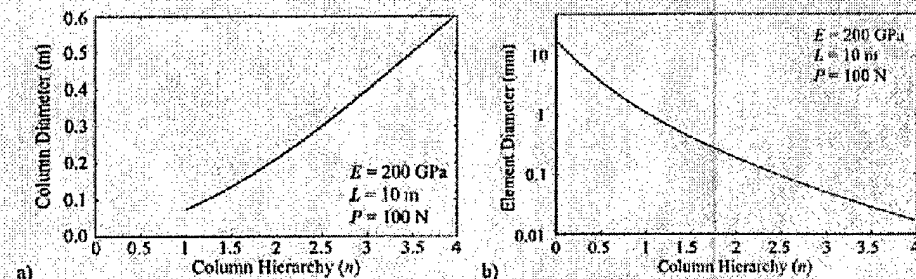

Figure 9: Column diameter (a) and smallest element diameter (b) as a function of hierarchy order.
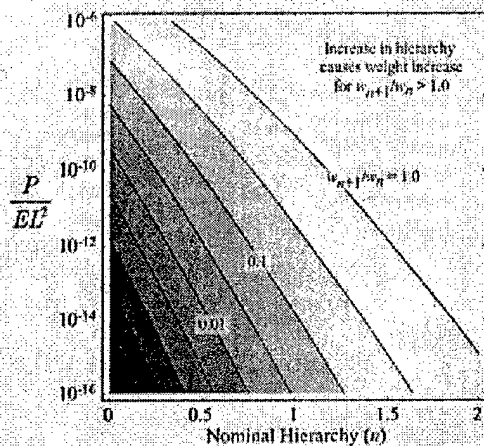
Figure 10: Ratio of $n + 1$ order structure weight per length to n order structure weight per length.

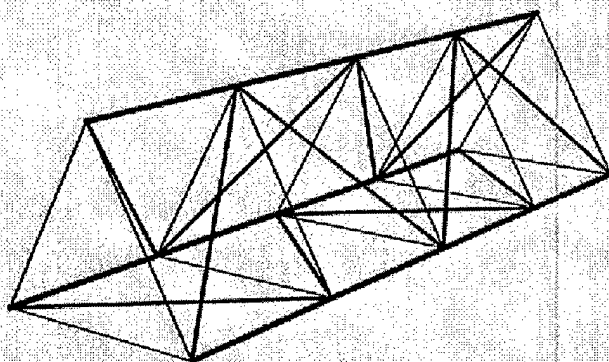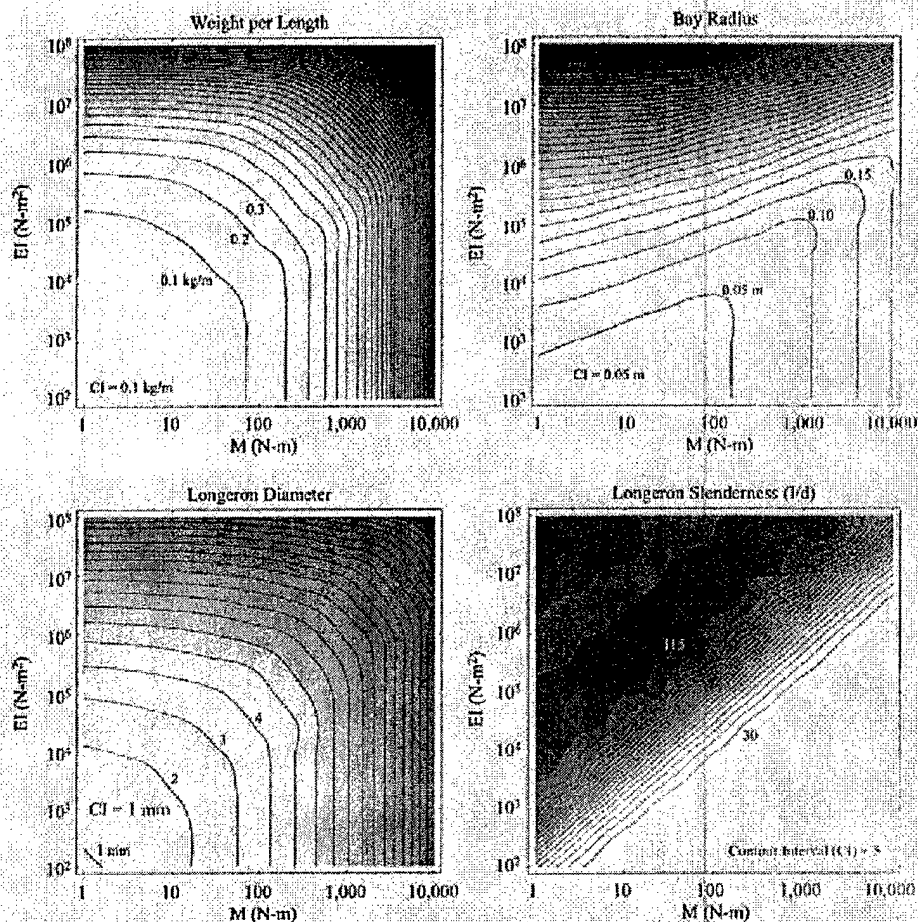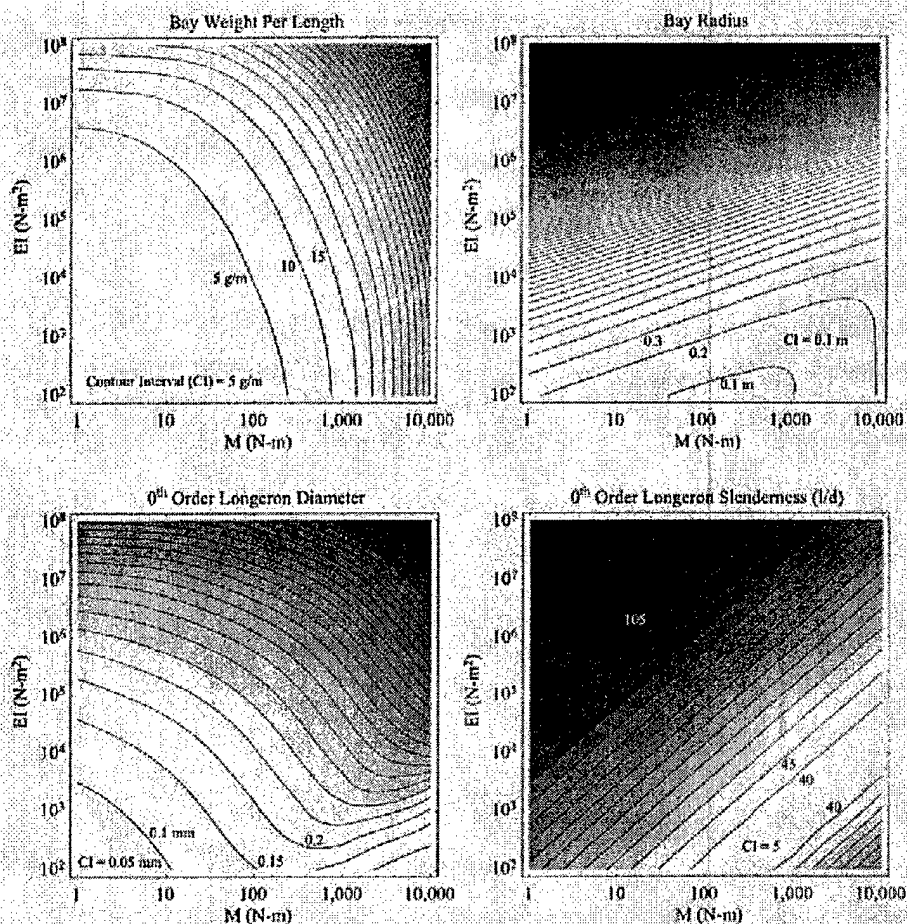Figure 11: Four bays of the triangular double-bay single-lacing truss.

Figure 12: 1ˢᵗ order optimization results (E = 200 GPa (29 Msi), ρ = 1660 kg/m³ (0.06 lb/in³)).

Unclassified

Figure 13: 2nd order optimization results (E = 200 GPa (29 Msi), ρ = 1660 kg/m³ (0.06 lb/in³)).
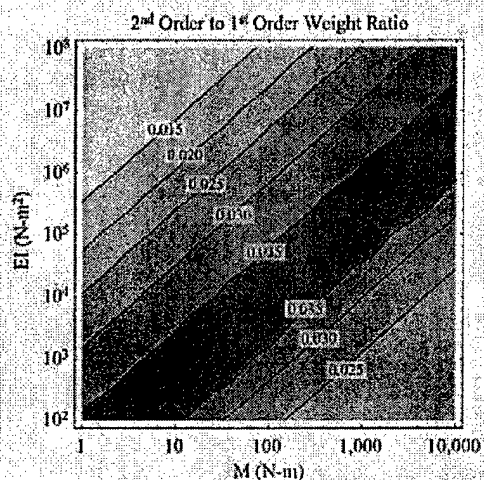
14
American Institute of Aeronautics and Astronautics

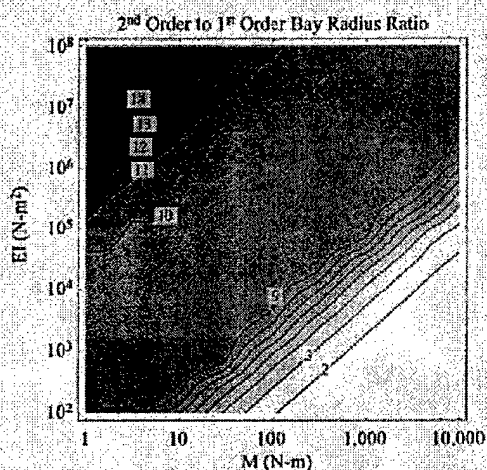**Figure 14: Ratio of 2nd order to 1st order truss weight per length optimization results (E = 200 GPa (29 Msi), ρ = 1660 kg/m³ (0.06 lb/in³)).**

Figure 15: Ratio of 2nd order to 1st order truss radius optimization results (E = 200 GPa (29 Msi), ρ = 1660 kg/m³ (0.06 lb/in³)).